Optimal Transmission Switching for Reducing Market Power Cost

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KTH Electrical Engineering

Degree project in
Electric Power Systems
Second Level,
Stockholm, Sweden 2012

XR-EE-ES 2012:016

ROYAL INSTITUTE
OF TECHNOLOGY

Optimal Transmission Switching for Reducing Market Power Cost

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A thesis submitted to the

Department of Electrical Power Systems

School of Electrical Engineering

KTH Royal Institute of Technology

in conformity with the requirements for

the degree of Master of Science

Stockholm, Sweden

September 2012

Abstract

The conventional transmission planning tends to focus exclusively on efficiency benefit, allowing cheaper remote generation to have priority dispatch over expensive local generation (least cost approach). Because of this nowadays deregulated markets face the problem that their systems affect the competitiveness of players, giving room for players to exercise market power.

The purpose of this study is to develop a mathematical model that quantifies the generation cost and reduces market power, by minimizing the social cost and restraining producers from withholding generation capacity. To do this, deterministic optimal transmission switching is proposed, together with a Worst-Nash Equilibrium (WNE) optimization, to quantify the social cost.

This study considers the transmission switch formulation based on the DC Optimal Power Flow (DCOPF) presented by Schmuel S. Oren as a Mixed-Integer linear Program (MIP). This formulation employs binary variables to represent the state of the transmission line. The effects of transmission switching with contingency analysis are also considered in the DCOPF formulation.

To include market power cost reduction in our problem, the social cost of the system is modeled considering WNE, which maximizes the social cost using linearization. The formulation includes strategic generators that might choose

to withhold some of their output and non-strategic generators. This under the condition that the profit of a portfolio with a strategic generator under Nash Equilibrium is always greater than the profit of a portfolio where the offers are constant.

A 14-node example system is studied where the efficiency benefits and competition benefits of transmission capacity by optimal transmission switching are considered. The results demonstrate that the utilization of the proposed method increase economic benefit and improves competitiveness in the electricity market.

Acknowledgments

I would like to show my gratitude to Professor Mohammad R. Hesamzadeh for trusting me with this master degree project, his guidance and motivation during this eight months and helping me to learn how to think more critical.

I also like to thanks Mahir Sarfati, Olga Galland, Faleye Omobobola and John Laury for their collaboration, suggestions and constructive critiques in a good working environment and friendship.

I would also like to give a special thanks to my family; Jose A. Noriega, Belle Odor de Noriega and Pedro J. Noriega for inspiring me and being a role model in my professional career and life. Last but not least to Remco Ammerlaan who without his support and encouragement I could not succeed in this and many other enterprises.

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Nomenclature

Indices

u	generator unit
l	transmission line
i	node of the network
n, m	nodes between line l
k	maximum generation capacity
p	portfolio
s	strategy
c	contingency

Sets

U	set of units in the system
L	set of transmission lines in the system
N	set of nodes in the system
K	set of constants
O	set of portfolios in the system
S	set of strategies

Model parameters

c_u	cost
$\underline{F}$	minimum transmission capacity
$\overline{F}$	maximum transmission capacity
$\overline{g}$	maximum generation capacity
d	demand
B	admitance
n_c	operation status of the line during contingency, 0 the line is contingency (disconnected)and 1 otherwise
M	arbitrary big number
$\mathbf{C}$	incidence matrix

Optimization model variables

SC	social cost
F	power flow in the transmission line
g	generation of each unit
$\widehat{g}$	maximum capacity offer to the market
n_l	operation status of the line, 0 the line is disconnected and 1 otherwise
π_p	profit of portfolio, under Nash equilibrium
$\pi_p^{p,s}$	profit of portfolio, without Nash equilibrium
θ	voltage angle of node i
P	local marginal price at node i
x	behavior status of the generating unit, 0 for strategic units and 1 for the non-strategic units

w	action of generating unit u, related to the capacity offer to the market
q	quantity produce of an identical product
$\mathcal{L}$	Lagrangian optimization function

Slackness variables

ρ_i	Slackness variable for the energy balance constraint
λ_l^{down}	Slackness variable for the lower limit of the power flow constraint
λ_l^{up}	Slackness variable for the upper limit of the power flow constraint
μ_u^{down}	Slackness variable for the lower limit of the generation limit constraint
μ_u^{up}	Slackness variable for the upper limit of the generation limit constraint
β_l^{down}	Slackness variable for the lower limit of the Kirchhoff's law constraint
β_l^{up}	Slackness variable for the upper limit of the Kirchhoff's law constraint
δ_i	Contingency slackness variable for the energy balance constraint
α_l^{down}	Contingency slackness variable for the lower limit of the power flow constraint
α_l^{up}	Contingency slackness variable for the upper limit of the power flow constraint
$\varepsilon_l^{\text{down}}$	Contingency slackness variable for the lower limit of the Kirchhoff's law constraint
$\varepsilon_l^{\text{up}}$	Contingency slackness variable for the upper limit of the Kirchhoff's law constraint

Binary variables for Complementary conditions

$b_l^{\text{low}-\lambda}$	Binary variable for the lower limit of the power flow constraint
$b_l^{\text{up}-\lambda}$	Binary variable for the upper limit of the power flow constraint
$b_l^{\text{low}-\mu}$	Binary variable for the lower limit of the generation limit constraint
$b_l^{\text{up}-\mu}$	Binary variable for the upper limit of the generation limit constraint
$b_l^{\text{low}-\beta}$	Binary variable for the lower limit of the Kirchhoff's law constraint

$b_l^{\mathrm{up}-\beta}$ Binary variable for the upper limit of the Kirchhoff's law constraint

$b_l^{\mathrm{low}-\alpha}$ Contingency binary variable for the lower limit of the power flow constraint

$b_l^{\mathrm{up}-\alpha}$ Contingency binary variable for the upper limit of the power flow constraint

$b_l^{\mathrm{low}-\varepsilon}$ Contingency binary variable for the lower limit of the Kirchhoff's law constraint

$b_l^{\mathrm{up}-\varepsilon}$ Contingency binary variable for the upper limit of the Kirchhoff's law constraint

Outputs

F power flow in the transmission line

g generation of each unit

$\widehat{g}$ maximum capacity offer to the market

n_l operation status of the line, 0 the line is disconnected and 1 otherwise

Chapter 1

Introduction

1.1 Background

The early electricity industry was designed as vertical integration; where generation, transmission and distribution created a "natural monopoly". This vertical-integrated market led to a lack of incentives to improve the system and reduce generation and distribution costs.

Over the last two decades industry analysts, regulators and policy makers have focused on an economically more efficient system. In December 1996 most of the European electricity markets had changed their policies to achieve a liberalized market, by allowing operating firms to compete freely in the energy market and prevent producers to no longer own the transmission and distribution network [4]. Liberalization decouples generation and transmission sectors; aiming to increase the efficiency, reduce electricity prices and create a more competitive market.

Electricity as a commodity presents special characteristics that must be considered: production must always meet the demand in a short period of time; demand is varying every second; and storage is very limited and expensive [5]. These considerations combined with an inelastic demand, transmission constrains and market fragmentation are incentives for producers to abuse the market equilibrium and alter the electricity price to increase their own profit. This ability of abusing market equilibrium is known as market power.

Market power is one of the main concerns of the deregulated markets nowadays; before liberalization vertical market power was exercised [2], now there is no more concern for that. Instead horizontal market power is a potential problem [6]; when suppliers collude with other suppliers in the market to gain the ability of altering the electricity price.

Evidence of horizontal market power abuse has been observed in several operating electricity markets, where the market price has increased up to 22% above competitive level [7]. Perfect competition price can be set as a benchmark to evaluate market power abuse, different indicators have been studied to quantify market power and its association with cost [8],[1].

There are different forms of market power that can be present; market dominance or local market power (due to transmission congestion). Scholars have studied transmission congestion in the past to recognize the presence of market power in the system [9]. Market dominance concerns strategic producers that are large enough to affect the price and maximize their profit. Strategies as Physical Withholding and Economical Withholding (bids) are implemented to alter the electricity price through horizontal mergers or by collusion. Strategic investment policies can also be implemented to maintain market dominance or increase profit [10].

Physical Withholding or Quantity Withholding occurs when a supplier intentionally decides to offer less generation capacity to the market than its maximum production capacity. Setting market price as the intersection between supply and demand curves [5], the quantity withheld (ΔQ_w) makes the supply curve shift to the left and a higher new price is set, increasing it from P^* to P^e as in figure (1.1).[1]

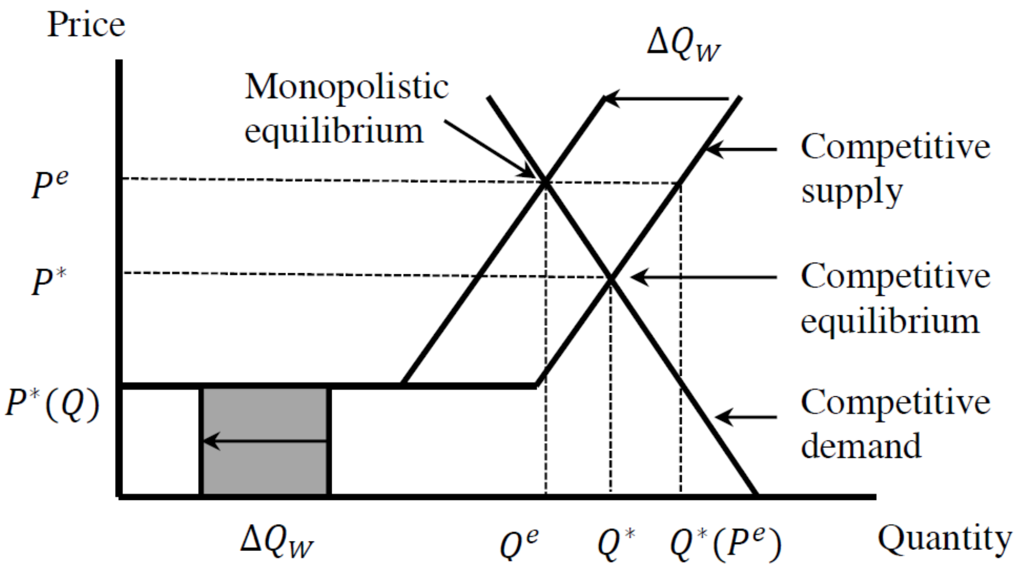

Figure 1.1: Withholding strategy and price-quantity outcome [1].

The main consequence of market power exercise is an increase of the generation cost (consequently electricity price) to a higher value than the marginal cost, usually referred to as Market Power Cost [8]. This means an increase of profit for producers and a lower efficiency in the market. The efficiency in the market is measured by its Social Welfare, the summation of producer's surplus and consumer's surplus [2]. The total surplus of the system is the total generation cost minus the variable cost of production. In a perfect competition market, the total surplus reaches its maximum and is considered to be an efficient market. In the case of market power, the social welfare reduces which indicates a lower efficiency. In figure 1.2 an example where in perfect competition the producer generation is

60TWh and its surplus is 6000MSEK, under market power the generating unit can offer less capacity (example 40TWh) until it reaches the maximum surplus of the producer (7000MSEK).

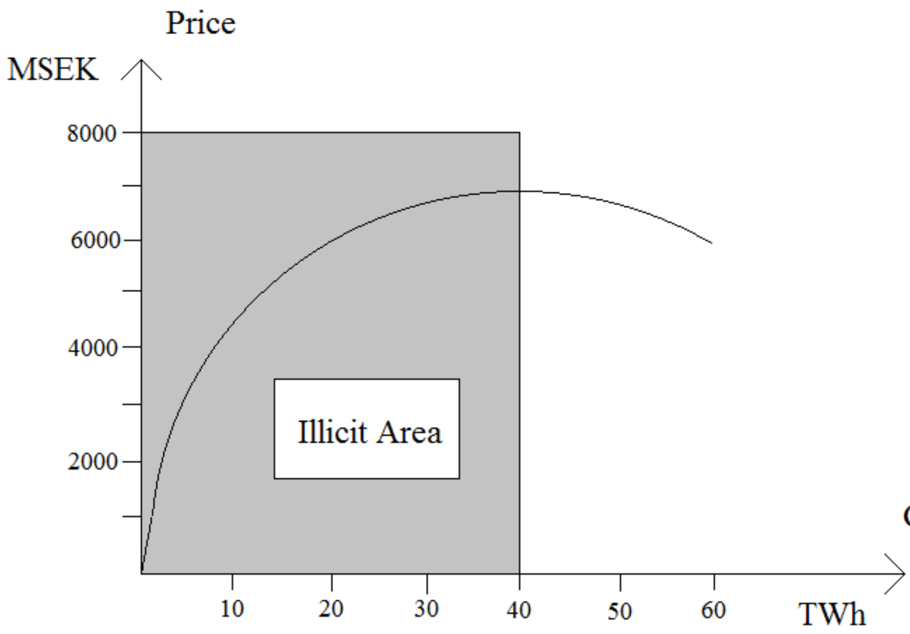

Figure 1.2: Withholding strategy and producers surplus variation [2].

This leads to another important consequence of market power exercise, which is the transfer of wealth from consumers to producers, as shown in figure 1.3. This transfer of wealth under market power directly affects consumers.

In general, perfect competition is assumed in a liberalized market, where no market power is exercised and suppliers bid according to their marginal cost. Nevertheless, in reality, considering the special characteristics of electricity, it is impractical to rely on this assumption. Therefore market power should be considered in the liberalized market and its regulation is of extreme importance to increase efficiency and minimize Social Cost.

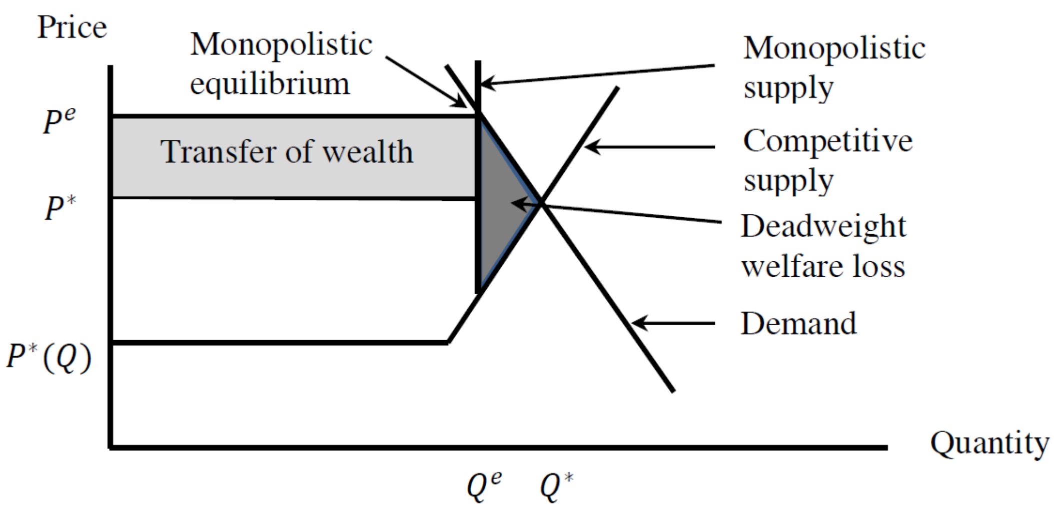

Figure 1.3: Transfer of wealth by the exercise of market power [1].

Traditionally, transmission switching analysis is not considered as an economic optimization; most of the studies were based only on system reliability criteria without any economic market analysis, where transmission elements were treated as non-dispatchable elements. Currently there is an agreement to develop a smarter electrical grid which is stronger, more flexible, more efficient and better to control. Optimal transmission switching has proven to be a reliable method of incorporating more controllability to the grid [11], and also creating economic benefit comparing to other methods such as generation unit rescheduling or load shedding [12]. Optimal transmission switching has been complemented with other co-optimizations ancillary services, generation topology and optimal unit commitment scheduling to improve economic efficiency under perfect competition assumption [11],[13].

Under the same assumption in a liberalized electricity market, effective optimal transmission switching has recently proved to be an important tool to improve overall market efficiency and reduce Social Cost [11], [14], [15] considering contingency constraints to prove the system remains stable and robust [16].

Previously market power strategic behavior for economical withholding has being model using Nash-Cournot concepts [17], and physical withholding has being reduce according to demand flexibility [18].

The increase of Social Welfare due to disconnection of a small number of lines in the system has been reported in [19]. Author of [19] suggests that by co-optimizing topology and a simple variation of the network by Transmission System Operators (TSO), market power can be reduced. An impact in market power regulation by transmission augmentation and additional transmission capacity has presented evidence of market power cost by regulating strategic behaviour [20],[21].

In the current electricity market structure, it is important to aim for a more efficient network, a smarter grid that can be more controllable [22]. A fair equilibrium for producers and consumers where there is no transfer of wealth must be procured.

The starting point of this thesis is the work carried out by [14] and [16] to formulate a transmission switching optimization problem to reduce Social Cost, and the work presented by [23] to include Extreme Nash-Equilibria concepts to model the strategic behavior of generating units (Market Power), where linearization techniques are implemented to formulate the problem as a single-stage mixed-integer linear program problem.

This thesis proposes an optimal transmission switching model to minimize Market Power Cost and therefore increase the competition and efficiency in the electricity market and reduce cost for society.

1.2 Problem Definition

Perfect competition has been a widely accepted assumption in electricity market studies. However if it is considered that the present deregulated electricity markets affect the competitiveness of players, by big producers being able to exercise market power, the main challenge of the coming environment for electricity market is reducing the cost associated with this behavior.

The idea of this project is to develop a mathematical model in order to: (1) expose how market power can increase social cost by withholding generation capacity, and (2) how optimal network configuration can reduce this cost and increase the competition of players by manipulating the connectivity of the transmission lines.

1.3 Objective

To accomplish this aim, reducing market power cost, two models are studied and integrated. The first model is based on the work made by [14] and [16], that considers traditional schemes of the electricity market with the assumption of perfect competition between players, which is not always the case. To have a better understanding of the real nature of the electricity market, imperfect competition must be included. The second model introduces market power behavior, using Game Theory concepts (Nash equilibrium) to model strategic units in the system, that can choose to withhold capacity offered to the electricity market.

Both models are integrated to have a system under market power conditions where transmission lines can be optimally switched to reduce the market power cost.

1.4 Resources/Tools used

The optimization problem is executed using the General Algebraic Modeling System (GAMS) platform. The deterministic optimization problem is solved with the CPLEX solver in GAMS. The incidence matrices and other sets of parameters are created in MATLAB and transferred to GAMS by using Comma-Separated Values (CSV) files. The code is run on a computer with Intel Xeon E5345 with a 2.33 GHz clocking frequency and 16 GB of RAM. Linear programming relaxation is employed to solve the optimization problem in GAMS.

Chapter 2

Formulation of the Mathematical Model

Nowadays it is important for the electricity market regulator to minimize any form of market power and to offer the consumer lower cost of generation. This thesis proposes the use of optimal transmission switching by Transmission System Operators to minimize market power behaviour of producers and lower market power cost. To minimize the Social Cost under market power, a DC Optimal Power Flow (DCOPF) model of the system [14] is considered.

This thesis presents an electricity market model that contemplates a market power formulation and a DCOPF formulation with optimal transmission switching, it also includes contingency analysis formulation. Both formulations are integrated in the model as a bi-level optimization problem [24].

This chapter explains the formulation followed to integrate two models. The first assuming perfect competition (DCOPF model) with transmission switching and the second considering market power behavior, based on the substitution of the equations of the DCOPF model by its equivalent optimality conditions, Karush-Kuhn-Tucker (KKT) conditions.

2.1 Assumptions

Following assumptions where made to simplified the model:

- Perfect information;
- No capacity limitations, units have enough installed capacity to supply the load;
- The load is not price sensitive;
- There are no losses in the system, $\sum_{i=1}^{N} G = \sum_{i=1}^{N} D$;
- All lines are available in the initial conditions;
- All generators are available;
- Non-strategic units offer 100% of the maximum capacity to the market;
- Each portfolio (owner) has only one strategic unit;
- Marginal pricing: price is set by the maximum production cost.

2.2 Model Description

A linearized bi-level optimization problem is proposed. First the DC Optimal Power Flow (DCOPF) model (with transmission switching) and Worst-Nash Equilibrium (WNE) model are formulated independently and in detail in section *2.2.1* and section *2.2.2* respectively. Finally the proposed model is formulated in section *2.2.3*, integrating the models described in section *2.2.1* and section *2.2.2* as a linearized bi-level optimization using Karush-Kuhn-Tucker (KKT) conditions.

2.2.1 Optimal Transmission Switching

The aim of this model is to minimize the generation cost of the system using optimal transmission switching, subject to physical constraints and Kirchhof laws for power flow. DC approximations are used in the system, so the model can be defined as a linear problem. This model is base under the assumption of perfect competition.

The objective function and constrains of the optimization problem are base on a DC Optimal Power Flow. Objective function and constraints are modified to include the switching of the transmission lines and the cost associated. The objective function of this optimization problem and its constraints are stated below [14]:

$$\text{Minimize} \qquad \sum_{u \in U} c_u g_u + \sum_{l=1}^{L} c_l \left(1 - n_l\right) \tag{2.1a}$$

Subject to:

$$0 \leq g_u \leq \overline{g_u} \qquad \forall u \in U \tag{2.1b}$$

$$\underline{F_l}\; n_l \leq F_l \leq \overline{F_l}\; n_l \qquad \forall l \in L \tag{2.1c}$$

$$g_i + d_i + F_l C_{l,i} = 0 \qquad \forall i \in N, \forall l \in N \tag{2.1d}$$

$$B_l \left(\theta_n - \theta_m\right) - F_l + \left(1 - n_l\right) M \geq 0 \qquad \forall l \in L \tag{2.1e}$$

$$B_l \left(\theta_n - \theta_m\right) - F_l - \left(1 - n_l\right) M \leq 0 \qquad \forall l \in L \tag{2.1f}$$

This formulation includes a new element: n_l, which is a binary variable, and it represents the connection status of each transmission line in the system. When a line l is connected, $n_l = 1$ and when it has been removed or opened, $n_l = 0$.

The objective function (2.1a) includes the cost of switching a transmission line. The constraints of the optimization problem are:

- The generation limits of each unit (2.1b);

- The power flow limits of the line (2.1c). All lines are modelled with the same maximum transmission capacity. The binary variable n_l enables the maximum capacity of the line when the line is connected and limits it to zero when the line is disconnected.

- The energy balance constraint (2.1d).

- Kirchhoff's law (2.1e),(2.1f), the mathematical function of this two equation is to accentuate the fact that a disconnected line has no power flow. If the line is connected, this variable will be zero and the formula represents the connection of the line between nodes n and m. The parameter M is an arbitrary large number, and it is introduced to influence how the system perceives the network configuration. If a transmission line is connected (nl=1) Kirchhoff Law represents the connection of the line between the nodes and the angles difference between θ_m and θ_n will encourage the power flow through the line. When the line is disconnected (nl=0) these two equations will no longer represent this angle difference between nodes (only a numerical affirmation) and completes the formulation to represent the disconnection of the line. This formulation will ensure no flow in an opened line [14].

Contingency Analysis

To ensure the system is able to sustain its reliability, contingency analysis is considered in the formulation. Additional constrains are added to the previous (2.1) to include the contingency status of the system:

$$\underline{F_{lc}}\, n_l\, n_c \leq F_{lc} \leq \overline{F_{lc}}\, n_l\, n_c \qquad \forall l \in_L \tag{2.2a}$$

$$g_i + d_i + F_{lc} C_{l,i} = 0 \qquad \forall i \in N, \forall l \in N \tag{2.2b}$$

$$B_l \left(\theta_{nc} - \theta_{mc}\right) - F_{lc} + \left(2 - n_l - n_c\right) M \geq 0 \qquad \forall l \in_L \tag{2.2c}$$

$$B_l \left(\theta_{nc} - \theta_{mc}\right) - F_{lc} - \left(2 - n_l - n_c\right) M \leq 0 \qquad \forall l \in_L \tag{2.2d}$$

A new element n_c (binary variable) is included in this constrains (2.2) to represent the connection status of each transmission line in the system during contingency [16]. Contingency is defined as the loss of one single element in the system, in this case a transmission line. When line l is contingency, $n_c = 0$ (for line l) and when is not contingency, $n_c = 1$.

Equations (2.1)-(2.2) formulate the model for DCOPF with Optimal Transmission Switching (OTS) and Contingency Analysis.

2.2.2 Worst-Nash Equilibrium

Assuming that generating units can behave strategically, choosing to withhold some of their outputs, we consider that market power is exercised by a supplier (owner of a strategic unit) when it is intentionally withholding its production capacity. This action will increase the cost of generation in the system and increase the profit of the unit. Perfect competition conditions for electricity market studies, where all units are non-strategic units (are assumed to bid competitively) is no longer an assumption, since some units are considered to be strategic in this model.

Game theory concepts are used in this project to model the strategic decision of the generating units, in particular Cournot Competition and Nash Equilibrium concepts.

The Cournot Competition concepts considered for this project include: (1) Few players (suppliers) and many strategies. (2) Strategies are define as the quantities (q) the players produce of an identical product. (3) The cost of production is $c_u q$ (with a constant marginal cost). (4) Players aime to maximize their profit [25].

A strategy profile S_p is a Nash Equilibrium if for each individual player, its choice s is the best response to the other players choices. Figure 2.1 shows the response curve of two players; $BR1(R2)$ reflects the best responses of player 1 (R1) according to player 2 responses (R2), the same for $BR2(R1)$. The strategy chosen by the each player where they intersect ($R1^*$ and $R2^*$ respectively) is the the Nash Equilibrium response for each of them.

Some considerations under Nash Equilibrium are: (1) Under Nash Equilibrium there is no individual incentive of each player to deviate from s if all actions of all players are hold constant. Only under Nash Equilibrium are no regrets for all players. (2) Everybody in the game believes that everybody else will also play their particular part in the Nash Equilibrium. (3) There can be many Nash Equilibriums for a problem. Problems can converge to a bad equilibrium for players base on decreasing the profits, and can also converge to a good equilibrium for the players base on increasing profits [26], [27].

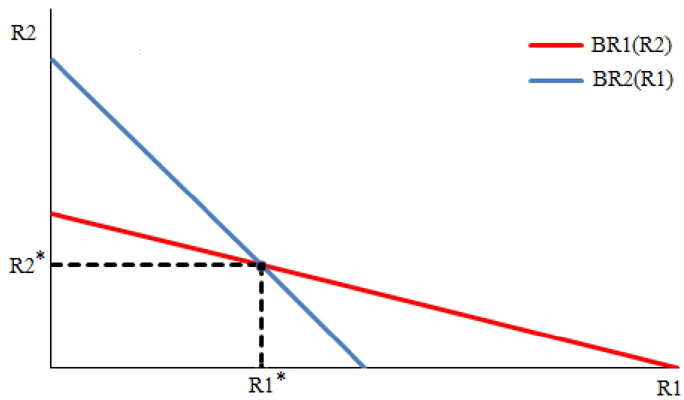

Figure 2.1: Nash equilibrium found by two players [3].

When all players reach their own strategy and thereby have maximized their own profit (π_p), the Nash Equilibrium has been reached [28]. In this study suppliers (producers) will act as players trying to find the Nash equilibrium to maximize their profit. Market power in the system is model as the Worst-Nash Equilibrium (WNE), defined as the Nash Equilibrium which maximizes the generation cost (Social Cost).

There will be strategic units and non-strategic units in the formulation. For this model, each producer that owns an strategic unit is define as to posses a portfolio for each strategic unit it owns. For simplicity it is assumed that each owner that behaves strategically may have maximum one portfolio. Then there will be as many portfolios as strategic units are. Where the strategic unit is able to exercise strategies considering Nash equilibrium.

The aim of this model (2.3) is to maximize the generation cost (Social Cost) of the system an obtain the Worst-Nash Equilibrium, subjected to the two main constraints. These constraints are:

- Non-strategic units will offer their maximum capacity to the system and strategic units can choose from a finite set of actions how much they will offer to the market.

- The profit of a portfolio under Nash Equilibria (π_p) is higher than the profit of a portfolio that does not considers Nash Equilibria for its dispatch ($\pi_p^{p,s}$).

$$\text{Maximize} \quad \sum_{u \in U} c_u g_u \tag{2.3a}$$

Subject to:

$$\widehat{g}_u = \sum_{k=1}^{K} \overline{g}_u x_{uk} w_k + w_0 \overline{g}_u \qquad \forall u \in U \tag{2.3b}$$

$$\pi_p \geq \pi_p^{p,s} \qquad u \in U \tag{2.3c}$$

Strategic and non-strategic units u are part of the set of generating units U, have a variable cost of production c_u and their production under Nash Equilibrium is g_u. Then the generation cost or Social Cost (SC) under Nash Equilibrium is define (2.3a). The offered capacity ($\widehat{g}_u$) of a strategic unit is determined by profit maximization. In (2.3b) $\overline{g}_u$ is the maximum capacity of the generating unit u, the binary variable x_{uk} indicates whether the unit is strategic $x_{uk} = 0$ or non-strategic $x_{uk} = 1$. The percentage of maximum capacity that a strategic unit will offer is defined by w_0. And the withheld capacity is defined by w_k so that $\sum_{k=1}^{K} w_k = 1 - w_0$. So when the unit is non-strategic $\sum_{k=0}^{K} w_k = 1$ and the unit will offer its maximum capacity (2.4). Each strategic unit is assumed to have potentially 4 levels of output, therefore each portfolio will have 4 different possible strategies; since a portfolio has only one strategic unit and no non-strategic units.

$$\widehat{g_u} = \begin{cases} \overline{g}_u w_0 & \text{if } x_{uk} = 0 \\ \overline{g}_u \sum_{k=0}^{K} w_k & \text{if } x_{uk} = 1 \end{cases} \tag{2.4}$$

It is stated before and again in (2.3c) that the profit of a portfolio under Nash Equilibrium π_u is greater than the profit of the same portfolio rather than when it is not considering the equilibrium strategies ($\pi_p^{p,s}$). We said that the same portfolio does not considers Nash equilibrium when we hold the offers of every other portfolio constant, and replace the offer of this portfolio with another strategic combination $s \in S_p$ [28]. If there were more than one strategic unit for portfolio, the profit of it would be the sum of the individual profits of each strategic units (π_u) as:

$$\pi_p = \sum_{u \in O(p)} \pi_u$$

For the proposed model the profit of a portfolio p, π_p, is the same as the profit of a unit u, π_u. The profit of a single unit can be defined as: $\pi_u = (P_i - c_u)g_u$, where P_i is the local marginal price at the node i, c_u is the variable cost of the unit and g_u is its respectively dispatch. This equation can also be expressed as $\pi_u = \mu_u^{\text{up}} \widehat{g_u}$, where μ_u^{up} is the Lagrange multiplier of the constraint of each unit so it cannot exceed its offer.

The expression for the profit of a unit π_u is not linear, so to linearize this expression two considerations are made [23]:

(1) If X and Y are positive variables in an optimization problem, b is a binary variable and M is a large number, the non-linear equation $XY = 0$ can be rewritten:

$$X_b + Y(1-b) = 0 \tag{2.5a}$$

Can be written as two simple linear constrains:

$$-M(1-b) \leq X \leq M(1-b) \tag{2.5b}$$

$$-Mb \leq Y \leq Mb \tag{2.5c}$$

(2) Considering if λ is a Lagrange multiplier and x is a decision variable, the multiplication of this two variables is not linear, supposing that x takes a set of finite values $x = \sum_{j=1}^{m} a_j b_j + a_0$ for constants $a_0, ..., a_m$ and m binary variables $b_1, ..., b_m$. Introducing a new variable c_j that satisfies: $c_j - \lambda b_j = 0$. This expression is of the form of (2.5a) and can be written as two simple linear constrains (2.5b),(2.5c).

Then the expression for the profit of a unit π_u is linearize introducing a variable $z_{uk} = \mu_u^{\text{up}} x_{uk}$, where x_{uk} is a binary variable. The equation z_{uk} is now linearized as three equations [23]:

$$z_{uk} - \mu_u^{\text{up}} \leq M\,(1 - x_{uk}) \tag{2.6a}$$

$$z_{uk} - \mu_u^{\text{up}} \geq -M\,(1 - x_{uk}) \tag{2.6b}$$

$$z_{uk} \leq M x_{uk} \tag{2.6c}$$

Introducing linearized equations (2.6) to the definition of the profit of a portfolio π_p:

$$\pi_p = \sum_{k=1}^{K} k_u z_{uk} w_k + \mu_u^{\mathrm{up}} w_0 k u \tag{2.7}$$

The model to obtain the Worst-Nash Equilibrium is formulated in (2.3)-(2.7). These equations are linearized, so the model can be defined as a linear problem.

2.2.3 Integrated Model of Optimal Transmission Switching and Worst-Nash Equilibrium

Nowadays it is important for the electricity market regulator to minimize any form of market power and to offer the consumers lower cost of generation. This thesis proposes the use of optimal transmission switching implemented by Transmission System Operators to minimize market power behaviour of producers and lower market power cost. A linearized bi-level optimization problem is proposed, WNE formulation and OTS formulation are integrated as a set of primary conditions and secondary conditions respectively. This integration is based on the substitution of the OTS formulation by its equivalent optimal conditions, also known as Karush-Kuhn-Tucker (KKT) conditions [29],[30]. The primary conditions are defined by the same formulation presented for the WNE model (2.3)-(2.7).

KKT conditions are developed for the DCOPF with Optimal Transmission Switching and Contingency Analysis model, these conditions will be defined as the secondary conditions of our integrated model. The inner dispatch problem minimizes the social cost, subject to constrains as in (2.1)-(2.2), except for (2.1b) which from now on will be redefine as $0 \leq g_u \leq \widehat{g_u}$ (2.8b) to include the strategic units behavior. The secondary conditions are defined as:

$$\text{Maximize} \qquad -\sum_{u \in U} c_u g_u + \sum_{l=1}^{L} c_l \left(1 - n_l\right) \tag{2.8a}$$

Subject to:

$$0 \leq g_u \leq \widehat{g_u} \qquad \forall u \in U \tag{2.8b}$$

$$\underline{F_l}\; n_l \leq F_l \leq \overline{F_l}\; n_l \qquad \forall l \in L \tag{2.8c}$$

$$g_i + d_i + F_l C_{l,i} = 0 \qquad \forall i \in N \tag{2.8d}$$

$$B_l \left(\theta_n - \theta_m\right) - F_l + \left(1 - n_l\right) M \geq 0 \qquad \forall l \in L \tag{2.8e}$$

$$B_l \left(\theta_n - \theta_m\right) - F_l - \left(1 - n_l\right) M \leq 0 \qquad \forall l \in L \tag{2.8f}$$

$$\underline{F_{lc}}\; n_l\; n_c \leq F_{lc} \leq \overline{F_{lc}}\; n_l\; n_c \qquad \forall l \in L \tag{2.8g}$$

$$g_i + d_i + F_{lc} C_{l,i} = 0 \qquad \forall i \in N \tag{2.8h}$$

$$B_l \left(\theta_{nc} - \theta_{mc}\right) - F_{lc} + \left(2 - n_l - n_c\right) M \geq 0 \qquad \forall l \in L \tag{2.8i}$$

$$B_l \left(\theta_{nc} - \theta_{mc}\right) - F_{lc} - \left(2 - n_l - n_c\right) M \leq 0 \qquad \forall l \in L \tag{2.8j}$$

First the Lagrangian function of the DCOPF with Optimal Transmission Switching and Contingency Analysis of (2.8) is defined as (2.9).Where ρ_i, λ_l^{down}, λ_l^{up}, μ_u^{down}, μ_u^{up}, β_l^{down}, β_l^{up}, δ_i, α_l^{down}, α_l^{up}, $\varepsilon_l^{\text{down}}$ and $\varepsilon_l^{\text{up}}$ are the Lagrange multipliers of the problem constraints.

$$
\begin{aligned}
\mathcal{L} = & - \sum_{u \in U} c_u g_u + \sum_{l=1}^{L} c_l \left(1 - n_l\right) \\
& + \rho_i \left(g_i + d_i + F_l C_{l,i}\right) \\
& + \sum_{l=1}^{L} \lambda_l^{\text{down}} \Big(\underline{F_l}\; n_l - F_l \Big) + \sum_{l=1}^{L} \lambda_l^{\text{up}} \Big(F_l - \overline{F_l}\; n_l \Big) \\
& + \sum_{u}^{U} \mu_u^{\text{down}} \Big(\underline{g_u} - g_u \Big) + \sum_{u}^{U} \mu_u^{\text{up}} \Big(g_u - \overline{g_u} \Big) \\
& + \sum_{l=1}^{L} \beta_l^{\text{down}} \Big(- B_l \left(\theta_n - \theta_m\right) + F_l - \left(1 - n_l\right) M \Big) \\
& + \sum_{l=1}^{L} \beta_l^{\text{up}} \Big(B_l \left(\theta_n - \theta_m\right) - F_l - \left(1 - n_l\right) M \Big) \\
& + \delta_i \left(g_i + d_i + F_{lc} C_{l,i}\right) \\
& + \sum_{l=1}^{L} \alpha_l^{\text{down}} \Big(\underline{F_{lc}}\; n_l\; n_c - F_{lc} \Big) \\
& + \sum_{l=1}^{L} \alpha_l^{\text{up}} \Big(F_{lc} - \overline{F_{lc}}\; n_l\; n_c \Big) \\
& + \sum_{l=1}^{L} \varepsilon_l^{\text{down}} \Big(- B_l \left(\theta_{nc} - \theta_{mc}\right) + F_{lc} - \left(2 - n_l - n_c\right) M \Big) \\
& + \sum_{l=1}^{L} \varepsilon_l^{\text{up}} \Big(B_l \left(\theta_{nc} - \theta_{mc}\right) - F_{lc} - \left(2 - n_l - n_c\right) M \Big)
\end{aligned}
\tag{2.9}
$$

The Dual feasibility conditions are:

$$\rho_i, \delta_i \rightarrow \text{free} \qquad \forall_i \in N \tag{2.10a}$$

$$\lambda_l^{\text{down}}, \lambda_l^{\text{up}} \geq 0 \qquad \forall_l \in L \tag{2.10b}$$

$$\mu_u^{\text{down}}, \mu_u^{\text{up}} \geq 0 \qquad \forall_u \in U \tag{2.10c}$$

$$\beta_l^{\text{down}}, \beta_l^{\text{up}} \geq 0 \qquad \forall_l \in L \tag{2.10d}$$

$$\alpha_l^{\text{down}}, \alpha_l^{\text{up}} \geq 0 \qquad \forall_l \in L \tag{2.10e}$$

$$\varepsilon_l^{\text{down}}, \varepsilon_l^{\text{up}} \geq 0 \qquad \forall_l \in L \tag{2.10f}$$

The Lagrangian function is formulated by setting each side of the constraints less/equal to zero, separating the lower boundary from the upper boundary conditions. Each of this conditions are multiplied by a Lagrange multiplier respectively. Finally, the objective function is added to the sum of these constraints as in (2.9).

The process is known as Lagrange relaxation, where these changes an unconstrain problem of minimization is obtain from a constrain problem with equality and inequality restrictions [29]. Now KKT conditions are derive from the Lagrangian.

The stationary conditions derived from the Lagrangian are defined as:

$$\frac{\partial \mathcal{L}}{\partial g_u} = -c_u + \rho_i - \mu_u^{\text{down}} + \mu_u^{\text{up}} + \delta_i = 0 \tag{2.11a}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_i} = \sum_{l=1}^{L} \beta_l^{\text{down}} \left(C_{i,l}\right)^T B_l - \sum_{l=1}^{L} \beta_l^{\text{up}} \left(C_{i,l}\right)^T B_l = 0 \tag{2.11b}$$

$$\frac{\partial \mathcal{L}}{\partial F_l} = \rho_i - \lambda_l^{\text{down}} + \lambda_l^{\text{up}} + \beta_l^{\text{down}} - \beta_l^{\text{up}} = 0 \tag{2.11c}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_{ic}} = \sum_{l=1}^{L} \varepsilon_l^{\text{down}} \left(C_{i,l}\right)^T B_l - \sum_{l=1}^{L} \varepsilon_l^{\text{up}} \left(C_{i,l}\right)^T B_l = 0 \tag{2.11d}$$

$$\frac{\partial \mathcal{L}}{\partial F_{lc}} = \delta_i - \alpha_l^{\text{down}} + \alpha_l^{\text{up}} + \varepsilon_l^{\text{down}} - \varepsilon_l^{\text{up}} = 0 \tag{2.11e}$$

The stationary conditions (2.11) are defined as the partial derivatives of the Lagrange function in respect to the variables of the secondary conditions and equal to zero. These variables are g_u, θ_i, F_l,θ_{ic} and F_{lc}.

Then complementary slackness conditions derived from the Lagrange function:

$$\sum_{l=1}^{L} \lambda_l^{\text{down}} \Big(\underline{F_l}\; n_l - F_l \Big) = 0 \qquad \forall l \in L \qquad (2.12\text{a})$$

$$\sum_{l=1}^{L} \lambda_l^{\text{up}} \Big(F_l - \overline{F_l}\; n_l \Big) = 0 \qquad \forall l \in L \qquad (2.12\text{b})$$

$$\sum_{u}^{U} \mu_u^{\text{down}} \Big(\underline{g_u} - g_u \Big) = 0 \qquad \forall u \in U \qquad (2.12\text{c})$$

$$\sum_{u}^{U} \mu_u^{\text{up}} \Big(g_u - \overline{g_u} \Big) = 0 \qquad \forall u \in U \qquad (2.12\text{d})$$

$$\sum_{l=1}^{L} \beta_l^{\text{down}} \Big(- B_l \left(\theta_n - \theta_m \right) + F_l - \left(1 - n_l \right) M \Big) = 0 \qquad \forall l \in L \qquad (2.12\text{e})$$

$$\sum_{l=1}^{L} \beta_l^{\text{up}} \Big(B_l \left(\theta_n - \theta_m \right) - F_l - \left(1 - n_l \right) M \Big) = 0 \qquad \forall l \in L \qquad (2.12\text{f})$$

$$\sum_{l=1}^{L} \alpha_l^{\text{down}} \Big(\underline{F_{lc}}\; n_l\; n_c - F_{lc} \Big) = 0 \qquad \forall l \in L \qquad (2.12\text{g})$$

$$\sum_{l=1}^{L} \alpha_l^{\text{up}} \Big(F_{lc} - \overline{F_{lc}}\; n_l\; n_c \Big) = 0 \qquad \forall l \in L \qquad (2.12\text{h})$$

$$\sum_{l=1}^{L} \varepsilon_l^{\text{down}} \Big(- B_l \left(\theta_{nc} - \theta_{mc} \right) + F_{lc} - \left(2 - n_l - n_c \right) M \Big) = 0 \qquad \forall l \in L \qquad (2.12\text{i})$$

$$\sum_{l=1}^{L} \varepsilon_l^{\text{up}} \Big(B_l \left(\theta_{nc} - \theta_{mc} \right) - F_{lc} - \left(2 - n_l - n_c \right) M \Big) = 0 \qquad \forall l \in L \qquad (2.12\text{j})$$

These complementary conditions only consider the constraints that are defined as less/equal than zero. They are the lower boundary and upper boundary of each constrain multiplied by their respective Lagrange multiplier and equal to zero.

Conditions in (2.12) are no-linear. To guaranty that the KKT conditions solve the problem and obtain the global optimal solution, these equations are reformulated as a Mix Integer Linear Programming problem (MILP) by introducing binary variables to linearize the problem as was estate above in (2.5). A set of two linear linear equations represent each equation of the complementary slackness condition:

$$-M\left(1-b_l^{\text{low}-\lambda}\right) \leq \left(\underline{F_l}\, n_l - F_l\right) \leq M\left(1-b_l^{\text{low}-\lambda}\right) \tag{2.13a}$$

$$-M\left(b_l^{\text{low}-\lambda}\right) \leq \lambda_l^{\text{low}} \leq M\left(b_l^{\text{low}-\lambda}\right) \tag{2.13b}$$

$$-M\left(1-b_l^{\text{up}-\lambda}\right) \leq \left(F_l - \overline{F_l}\, n_l\right) \leq M\left(1-b_l^{\text{up}-\lambda}\right) \tag{2.13c}$$

$$-M\left(b_l^{\text{up}-\lambda}\right) \leq \lambda_l^{\text{up}} \leq M\left(b_l^{\text{up}-\lambda}\right) \tag{2.13d}$$

$$-M\left(1-b_u^{\text{low}-\mu}\right) \leq \left(\underline{g_u} - g_u\right) \leq M\left(1-b_u^{\text{low}-\mu}\right) \tag{2.13e}$$

$$-M\left(b_u^{\text{low}-\mu}\right) \leq \mu_u^{\text{low}} \leq M\left(b_u^{\text{low}-\mu}\right) \tag{2.13f}$$

$$-M\left(1-b_u^{\text{up}-\mu}\right) \leq \left(g_u - \overline{g_u}\right) \leq M\left(1-b_u^{\text{up}-\mu}\right) \tag{2.13g}$$

$$-M\left(b_u^{\text{up}-\mu}\right) \leq \mu_u^{\text{up}} \leq M\left(b_u^{\text{up}-\mu}\right) \tag{2.13h}$$

$$-M\left(1-b_l^{\text{low}-\beta}\right) \leq \Big(-B_l\left(\theta_n-\theta_m\right) + F_l - \left(1-n_l\right)M\Big) \leq M\left(1-b_l^{\text{low}-\beta}\right) \tag{2.13i}$$

$$-M\left(b_l^{\text{low}-\beta}\right) \leq \beta_l^{\text{low}} \leq M\left(b_l^{\text{low}-\beta}\right) \tag{2.13j}$$

$$-M\left(1-b_l^{\mathrm{up}-\beta}\right) \leq \Big(B_l\left(\theta_n-\theta_m\right)-F_l-\left(1-n_l\right)M\Big) \leq M\left(1-b_l^{\mathrm{up}-\beta}\right) \tag{2.13k}$$

$$-M\left(b_l^{\mathrm{up}-\beta}\right) \leq \beta_l^{\mathrm{up}} \leq M\left(b_l^{\mathrm{up}-\beta}\right) \tag{2.13l}$$

$$-M\left(1-b_l^{\mathrm{low}-\alpha}\right) \leq \left(\underline{F_{lc}}\, n_l\, n_c - F_{lc}\right) \leq M\left(1-b_l^{\mathrm{low}-\alpha}\right) \tag{2.13m}$$

$$-M\left(b_l^{\mathrm{low}-\alpha}\right) \leq \alpha_l^{\mathrm{low}} \leq M\left(b_l^{\mathrm{low}-\alpha}\right) \tag{2.13n}$$

$$-M\left(1-b_l^{\mathrm{up}-\alpha}\right) \leq \left(F_{lc} - \overline{F_{lc}}\, n_l\, n_c\right) \leq M\left(1-b_l^{\mathrm{up}-\alpha}\right) \tag{2.13o}$$

$$-M\left(b_l^{\mathrm{up}-\alpha}\right) \leq \alpha_l^{\mathrm{up}} \leq M\left(b_l^{\mathrm{up}-\alpha}\right) \tag{2.13p}$$

$$-M\left(1-b_l^{\mathrm{low}-\varepsilon}\right) \leq \Big(-B_l\left(\theta_{nc}-\theta_{mc}\right)+F_{lc}-\left(2-n_l-n_c\right)M\Big) \leq M\left(1-b_l^{\mathrm{low}-\varepsilon}\right) \tag{2.13q}$$

$$-M\left(b_l^{\mathrm{low}-\varepsilon}\right) \leq \varepsilon_l^{\mathrm{low}} \leq M\left(b_l^{\mathrm{low}-\varepsilon}\right) \tag{2.13r}$$

$$-M\left(1-b_l^{\mathrm{up}-\varepsilon}\right) \leq \Big(B_l\left(\theta_{nc}-\theta_{mc}\right)-F_{lc}-\left(2-n_l-n_c\right)M\Big) \leq M\left(1-b_l^{\mathrm{up}-\varepsilon}\right) \tag{2.13s}$$

$$-M\left(b_l^{\mathrm{up}-\varepsilon}\right) \leq \varepsilon_l^{\mathrm{up}} \leq M\left(b_l^{\mathrm{up}-\varepsilon}\right) \tag{2.13t}$$

Recalling that the primary conditions are defined by the same formulation presented for the WNE model (2.3)-(2.7), the integration of the secondary conditions to the primary conditions in the bi-level problem is now complete by defining the secondary conditions as (2.8), (2.11) and (2.13).

The complete optimization problem is formulated in (2.14), where the objective function is to find the the maximum Social Cost under Nash equilibrium and minimize it by implementing Optimal Transmission Switching. The overall problem is a Mixed-Integer Linear Program:

$$\text{Maximize} \qquad (2.8a) \tag{2.14a}$$

$$\text{Subject to:} \qquad (2.3b) - (2.7), (2.8b) - (2.8j), (2.11), (2.13) \tag{2.14b}$$

Chapter 3

IEEE 14 Bus Test Case, Results and Discussion

For a better understanding the IEEE fourteen node example system is modified and used to illustrate the proposed model. Data from the system was obtained from the University of Washington Power System Test Case Archive [31]. Both transmission switching in perfect competition and under market power are applied to the modified IEEE 14-node example system and studied in detail. The deterministic optimization problem is solved with the CPLEX solver in General Algebraic Modeling System (GAMS) platform [32]. The incidence matrices and other sets of parameters are created in MATLAB and transferred to GAMS by using Coma-Separated Values (CSV) files. The code is run on a computer with Intel Xeon E5345 with a 2.33 GHz clocking frequency and 16 GB of RAM. Linear programming relaxation is employed to solve the optimization problem in GAMS.

3.1 Modified IEEE 14 Node Example System

The modified IEEE 14-node example system is illustrated in figure 3.1, this example system has fourteen nodes and twenty power lines. The generation costs are assumed to be linear, resistance of the line and shunt capacitance are zero, maximum transmission capacity of the lines is set to 20MW, losses and reactive power are ignored. The example system in figure 3.1 is modified to have three portfolios, each with one strategic unit that has four possible levels of output. The strategic units are u_3, u_5 and u_6 (shadow units) and they belong to Portfolio 1, Portfolio 2 and Portfolio 3 respectively. The characteristics of the generators is presented in table 3.1, table 3.2 shows the load data and transmission lines data is presented in table 3.3. Modifications from the IEEE test system are made for the generators data, also synchronous condensers (bus 3, bus 6 and bus 8) are not considered for this study.

Table 3.1: Generator data.

ID	Bus	Unit Size [MW]	Cost of generation [$/MWh]
u_1	1	30	30
u_2	2	200	20
u_3	3	150	100
u_4	3	100	10000
u_5	13	100	5000
u_6	6	100	100
u_7	3	100	150
u_8	14	100	100

3.2 Results and Discussion

3.2.1 The Traditional Model for Perfect Competition vs. Nash Equilibria Model

The traditional model for perfect competition is a Mixed Integer Linear Programming problem presented in (2.1) as Optimal Power Flow formulation, coded using

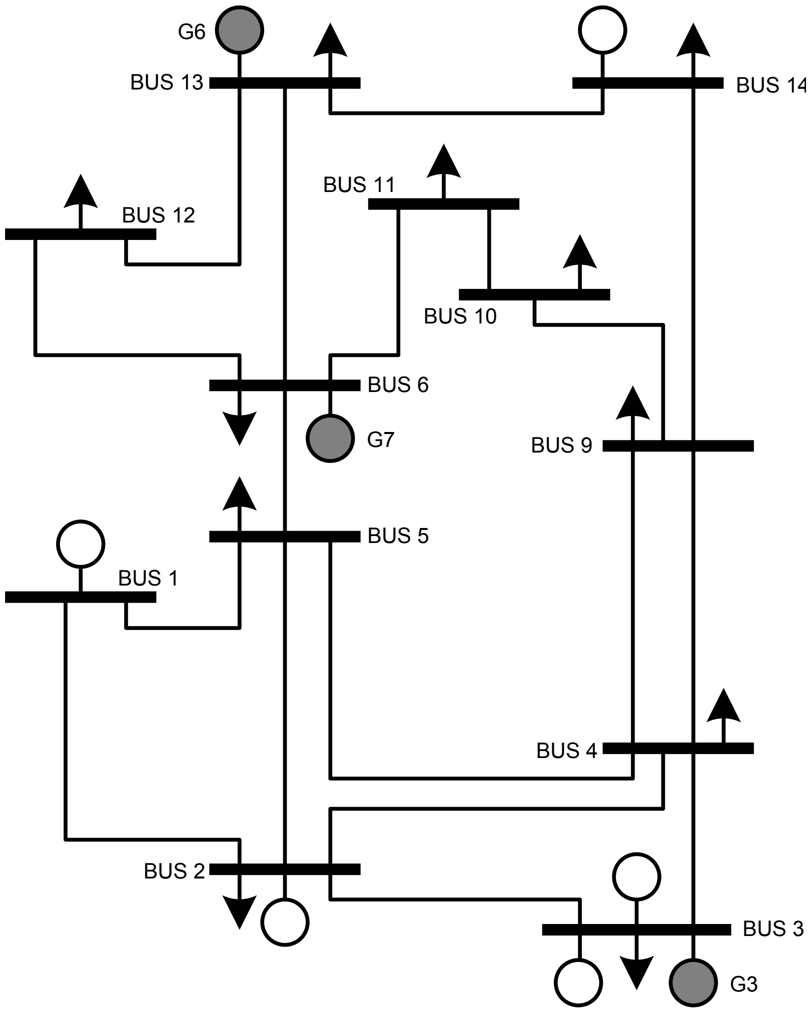

Figure 3.1: Single line diagram of the 14 node example system.

the GAMS platform and solved with the CPLEX solver. The optimal generation dispatch for the 14-node example system in perfect competition is shown in table 3.4. Transmission switching under perfect competition (A transmission element open) will result in SC increase, therefor the optimal solution is no transmission element open (minimum SC). When Nash equilibrium is considered in the example system with no transmission switching in the lines, the results are referred to as Base Case study. The MILP programming dispatch results are obtained and

Table 3.2: Load data.

Bus	Total Demand [MW]
2	21.7
3	94.2
4	47.8
5	7.6
6	11.2
9	29.5
10	9
11	3.5
12	6.1
13	13.5
14	14.9

presented in table 3.5.

Under traditional perfect competition, u_3 offers its true marginal cost to the market. The same unit under Nash equilibrium withholds about 93.75% of its real capacity. The Base Case where units are allowed to have strategic behavior by withholding capacity, shows a drastic increase in social cost from \$20688 MWh to \$60117 MWh, which is almost 290%.

This study shows the impact that strategically behaving generators can cause and how important it is to control its disposition to physical withholding. It also provides with a better understanding of how important it is to consider market power when economic considerations are studied in the system and social cost is to be minimized.

Table 3.3: Line data.

ID	From Bus	To Bus	X_{ID} [Ω]
L_1	1	2	0.05917
L_2	1	5	0.22304
L_3	2	3	0.19797
L_4	2	4	0.17632
L_5	2	5	0.17388
L_6	3	4	0.17103
L_7	4	5	0.04211
L_8	4	7	0.20912
L_9	4	9	0.55618
L_{10}	5	6	0.25202
L_{11}	6	11	0.19890
L_{12}	6	12	0.25581
L_{13}	6	13	0.13027
L_{14}	7	8	0.17615
L_{15}	7	9	0.11001
L_{16}	9	10	0.08450
L_{17}	9	14	0.27038
L_{18}	10	11	0.19207
L_{19}	12	13	0.19988
L_{20}	13	14	0.34802

3.2.2 The Proposed Model for Reducing Market Power Cost

In contrast with the Base Case for the Nash Equilibrium model, the proposed model for reducing market power cost (2.14) applied to the 14-node example system can be used to obtain an optimal planning for switching several lines in the system (lines 4-9 and 13-14). After opening the transmission lines suggested by the program, the system's network is shown as in figure 3.2.

Table 3.4: Optimal dispatch for perfect competition.

u	g [MW]	Cost [$/h]
u_1	30	900
u_2	38.89	777.8
u_3	101.762	10176.2
u_4	0	0
u_5	0	0
u_6	46.828	4682.8
u_7	0	0
u_8	41.52	4152
Total	259	**20689**

Table 3.5: Optimal dispatch under Nash equilibrium (base case).

u	g [MW]	Cost [$/h]
u_1	30	900
u_2	44.441	888.82
u_3	9.375	937.5
u_4	2.203	22030
u_5	2.666	13330
u_6	25	2500
u_7	100	15000
u_8	45.316	4531.6
Total	259	**60118**

Table 3.6: Optimal dispatch under Nash equilibrium (switched line).

u	g [MW]	Cost [$/h]
u_1	0.0007	0.021
u_2	101.699	2033.98
u_3	84	8400
u_4	0	0
u_5	0	0
u_6	38.4	3840
u_7	0	0
u_8	34.9	3490
Total	259	**17764**

Table 3.7: Generation capacity offer to the market in the base case.

u	$\overline{g}$ [MW]	$\widehat{g}$ [MW]
u_1	30	30
u_2	200	200
u_3	150	84
u_4	100	100
u_5	100	32.15
u_6	100	38.4
u_7	100	100
u_8	100	100

The new topology of the system changes the power flow of the lines as shown in table 3.8. It shows an increase of 233.3% in congested lines. When a fault occurs in one of the transmission lines that are congested before switching, the proposed model shows the system remains stable and transmission lines are optimally switched off to minimize the social cost.

The results show a significantly lower Social Cost compared to the Base Case,

Table 3.8: Power flow before/after switching.

	F_L [MW]	
ID	Before	After
L_1	14.199	-19.9
L_2	15.801	20
L_3	1.502	20
L_4	20	20
L_5	15.437	20
L_6	18.88	9.8
L_7	20	20
L_8	7.04	-
L_9	4.04	2
L_{10}	3.638	12.4
L_{11}	10.92	20
L_{12}	3.53	3.525
L_{13}	2.988	16.075
L_{14}	0	0
L_{15}	7.04	-
L_{16}	12.5	12.5
L_{17}	-20	-20
L_{18}	-3.5	-3.5
L_{19}	-2.57	-2.575
L_{20}	10.416	-

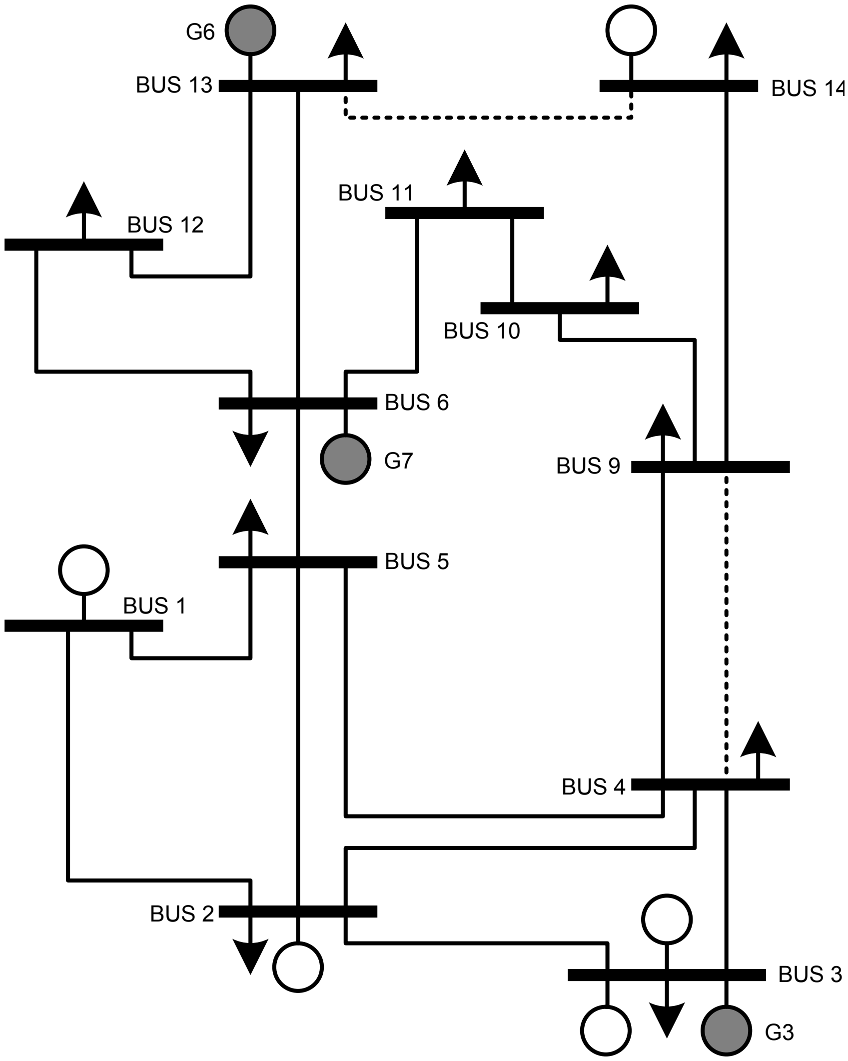

Figure 3.2: Single line diagram of the 14 node example system after switching.

table 3.9, with a reduction of 70.45%. This proposed model encourages strategic units (u_3, u_5, u_6) to be more competitive and offer more capacity to the market, as it is shown in table 3.10. Hereby the total withheld capacity of the system has been reduced from 70.4% to 55.8% (figure 3.3) of the installed capacity in strategic generators, which is an increase of 51.43MW capacity offered to the market. The strategic units u_3 and u_6 are the major contributors to reducing the Social Cost by

Table 3.9: Social cost.

# lines switched	Open Lines	SC [$/MWh]
0	-	60118
2	Bus 4-9, Bus 13-14	17764

Table 3.10: Generation capacity offer before/after switching.

u	$\overline{g}$ [MW]	$\widehat{g}_u$ [MW]	
		before	after
u_1	30	30	30
u_2	200	200	200
u_3	150	9.375	84
u_4	100	100	100
u_5	100	68.75	32.15
u_6	100	25	38.4
u_7	100	100	100
u_8	100	100	100

clearly reducing their withheld strategy, bidding more competitively and offering a capacity closer to their maximum as is shown in figure 3.4 and figure 3.5.

Optimal Transmission Switching reduces the total withholding, but looking closely at table 3.10 we can see that u_5 instead of reducing its own withholding, the proposed model has given it incentive to exercise more market power (figure 3.6); nevertheless the total Social Cost still decreases and hence society benefits from it.

The switching of these lines (lines 4-9 and 13-14) jointly give the highest social cost reduction. If there is a limitation on the number of lines that are allowed to switch and it is lower than the optimal solution, the Social Cost will still be reduced and lower than the Base Case social cost, but will be higher than the

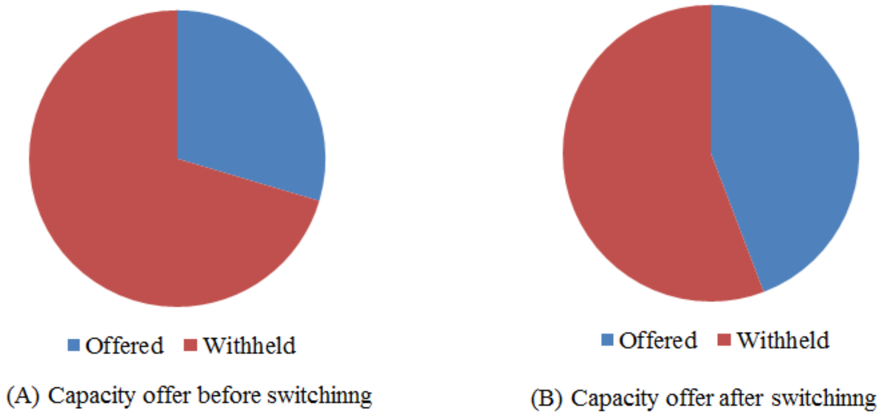

Figure 3.3: Impact of optimal transmission switching in withholding reduction.

optimal solution.

It has been proven that Optimal Transmission Switching can be used as one of many policies to increase Competition Benefit by minimizing market power cost. These results show the high impact of switching a small number of lines in the system on the total operational cost. It is important to notice that the proposed method is made for the systems that were designed or expanded by only taking into consideration the excess demand in the system, therefore the Social Cost are not considered during planning, which are most of the systems in operation today.

3.2.3 Contingency Analysis

To ensure the system is able to sustain its reliability under the proposed model for reducing Market Power cost, contingency analysis is considered in formulation (2.14). To represent the connection status of each transmission line during contingency, the parameter n_c (binary variable) is introduced in the formulation. The variable tests each line as a contingency element and evaluate its effects on

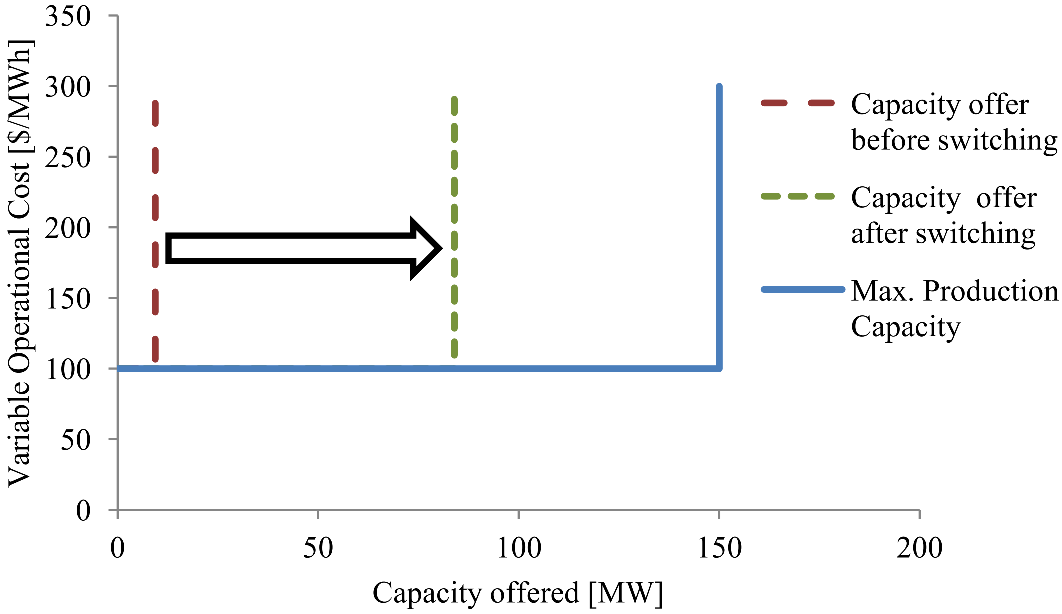

Figure 3.4: Marginal Cost Curve of u_3.

the system reliability including Optimal Transmission Switching to reduce market power cost.

When contingency is defined for a transmission lined, a new network topology is set. For this topology the program suggests an optimal planning for switching several lines in order to minimize the social cost. The proposed model considering contingency analysis is applied to the 14-node example system.

Contingency for each line is studied and results are presented in table 3.11. For most of the contingency cases transmission lines are optimally switched off to minimize the social cost, for these contingency cases the system remains stable. During the contingency analysis the Social Cost variate for different cases, some of them present an increase up to 9% of the social cost (for contingencies in line 2, line 3 and line 5 respectively) compared to the social cost for optimal transmission

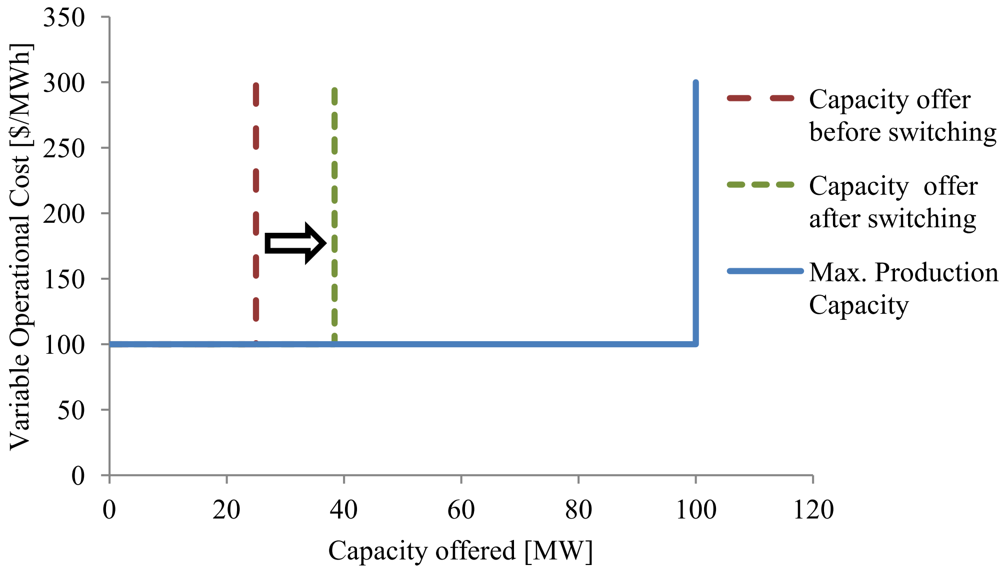

Figure 3.5: Marginal Cost Curve of u_6.

switching under Nash Equilibria when there is no contingency ($17764 MWh).

Observing table 3.11 the optimal transmission lines switched to reduce SC under contingency for the feasible results are mainly the same as the ones proposed in section 3.2.2. Then for these contingency scenarios it can be concluded that the system can sustain its reliability under the proposed model for reducing social cost.

Nevertheless this conclusions can not be generalized for all scenarios, table 3.11 shows infeasible results for several contingency conditions (35% of the contingency cases are infeasible). An infeasible solution stands for: there is no solution that fulfills all the constrains, the problem has no solution [5]. These infeasibilities do not necessarily reflect the stability of the system. There are two main reasons to which these infeasibility results can be attributed: (1) There is no Nash Equilibrium found in the system, there is no strategy that can be arranged between the

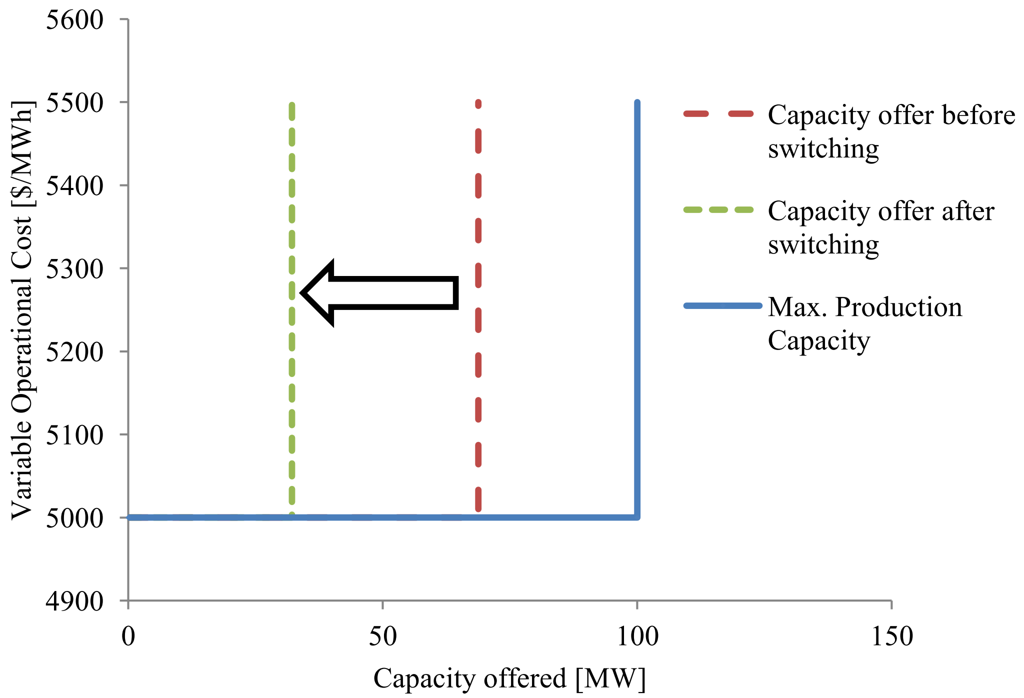

Figure 3.6: Marginal Cost Curve of u_5.

players (producers). (2) The mathematical model implemented is too simple to reach the solution.

Therefore taking into account all these considerations, the proposed model (optimal transmission switching to reduce market power cost) presents promising results to sustain the reliability of the system after a fault has occur.

Table 3.11: Contingency analysis (SC & OTS under NE)

Contingency	Open lines	SC [$/MWh]
L_1	L_8,L_{14},L_{15},L_{20}	17964
L_2	L_1,L_{14}	19364
L_3	L_8,L_{14},L_{15},L_{20}	19364
L_4	-	infeasible
L_5	L_{14},L_{20}	19364
L_6	-	infeasible
L_7	-	infeasible
L_8	L_{14},L_{15},L_{20}	17764
L_9	L_{14},L_{20}	17764
L_{10}	L_8,L_{12},L_{14},L_{15},L_{20}	18756
L_{11}	-	infeasible
L_{12}	L_8,L_{14},L_{15},L_{20}	17764
L_{13}	L_8,L_{14},L_{15}	17764
L_{14}	L_8,L_{15},L_{20}	17764
L_{15}	L_8,L_{14},L_{20}	17764
L_{16}	-	infeasible
L_{17}	-	infeasible
L_{18}	-	infeasible
L_{19}	L_8,L_{14},L_{15},L_{20}	17764
L_{20}	L_8,L_{14},L_{15}	17764

Chapter 4

Conclusions and Future Work

4.1 Conclusion

Nowadays deregulated markets face the problem that their transmission systems had been designed with mainly efficiency considerations, this affects the competitively of players, giving room for producer to exercise horizontal market power.

A mathematical model was developed to quantify the generation cost and reduce market power. Horizontal market power was reduced by restraining producers from withholding their generation capacity.

Optimal transmission switching is proposed with a Worst-Nash Equilibrium (maximizes Social Cost) linear optimization and tested on a 14-node example system.

The proposed mathematical formulation is based on game theory concepts (Nash equilibrium); the formulation includes strategic generators that might choose to withhold some of their output and non-strategic generators that offer their maximum production capacity to the market.

The proposed method has improved competition, reducing the total withheld capacity of the system from 70.4% to 55.8% of the installed capacity in strategic generators. The system increases 51,43MW of generation capacity offer to the market. Results demonstrate that the Social Cost of the system has reduced up to 70.45%, increasing the efficiency of the market.

Including optimal transmission switching increases the economic benefit in systems designed with only reliability and efficiency planning considerations. This thesis gives a glance of the impact and how important it is for society that transmissions planning considers not only meeting the demand but also reducing the cost for society.

4.2 Future Work

The proposed model considers a transmission switching formulation based on a DC Optimal Power Flow. Further studies consider the use of an AC Optimal Power Flow formulation to measure the impact of the transmission switching in the voltage levels, reactive power and transient stability.

A multi-period model is recommended to investigate the variations that are observed in the demand during the different seasons in a year and their impact in reducing market power.

Introducing optimal transmission switching in large scale can present a challenge due to large computational time. To make this feasible, the impact of optimal transmission switching in specific areas of the system should be studied closely to determine if they can affect the overall efficiency of the network.

A more detailed and stronger reliability criteria analysis must be carried and ensure reliability of the system after transmission switching. One consideration is that the example system was not congested before optimal transmission switching implementation; congested network previous switching must be taken into account to see how transmission switching (to regulate market power) can jeopardize the system reliability.

Future work includes optimal control of power flow capacity through the transmission lines to minimize market power cost by using power electronics in the transmission lines.

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Appendix A

GAMS Code

```
* Modelling Optimal Transmission switching and Extremal Nash Equilibria in Wh»
olesale Electricity Markets
* Using a Single Stage Mixed Integer Linear Program
*
* This version: 1.0 (29 July 2012)

 Sets
   p portfolios owning strategic generating units
      /p1*p3/
   n set of connection nodes in the system
      /n1*n14/
   u generating units
      /u1*u11/
   owns(p,u) ownership relation between portfolios and units
      /p1.u3, p2.u6, p3.u7/
   nd(n,u) connection node of each generating unit
      /n1.u1, n2.u2, n3.u3, n3.u4, n8.u5, n13.u6, n6.u7, n3.u8, n14.u9, n4.u1»
0, n9.u11/
   l transmission links
      /l1*l20/
   nt(n,l) conection node of each line
     /n1.l1, n2.l1, n1.l2, n5.l2, n2.l3, n3.l3, n2.l4, n4.l4, n2.l5, n5.l5, n»
3.l6, n4.l6, n4.l7, n5.l7, n4.l8, n7.l8, n4.l9, n9.l9, n5.l10, n6.l10, n6.l11»
, n11.l11,
      n6.l12, n12.l12, n6.l13, n13.l13, n7.l14, n8.l14, n7.l15, n9.l15, n9.l1»
6, n10.l16, n9.l17, n14.l17, n10.l18, n11.l18, n12.l19, n13.l19, n13.l20, n14»
.l20 /
   ss range of scenarios to be considered in a year /ss1/
;
 Parameters
   cost(u) variable cost of generating unit u dollars per MWh
       /u1 30, u2 20, u3 100, u4 10000, u5 5000, u6 100, u7 150, u8 100/

   cap(u,ss) maximum production capacity for unit u in MW
        /u1.ss1 30, u2.ss1 200, u3.ss1 150, u4.ss1 100, u5.ss1 100, u6.ss1 10»
0, u7.ss1 100, u8.ss1 100/

   F(l,ss) flow limits
        /l1.ss1 20, l2.ss1 20, l3.ss1 20, l4.ss1 20, l5.ss1 20, l6.ss1 20, l7»
.ss1 20, l8.ss1 20, l9.ss1 20, l10.ss1 20, l11.ss1 20, l12.ss1 20, l13.ss1 20»
, l14.ss1 20, l15.ss1 20, l16.ss1 20, l17.ss1 20, l18.ss1 20, l19.ss1 20, l20»
.ss1 20/

   Fc(l,ss) flow limits
        /l1.ss1 20, l2.ss1 20, l3.ss1 20, l4.ss1 20, l5.ss1 20, l6.ss1 20, l7»
.ss1 20, l8.ss1 20, l9.ss1 20, l10.ss1 20, l11.ss1 20, l12.ss1 20, l13.ss1 20»
, l14.ss1 20, l15.ss1 20, l16.ss1 20, l17.ss1 20, l18.ss1 20, l19.ss1 20, l20»
.ss1 20/

   C(l,n) incidence matrix
        /l1.n1  -1,  l1.n2   1,
        l2.n1   -1,  l2.n5   1,
        l3.n2   -1,  l3.n3   1,
        l4.n2   -1,  l4.n4   1,
        l5.n2   -1,  l5.n5   1,
        l6.n3   -1,  l6.n4   1,
        l7.n5   -1,  l7.n4   1,
        l8.n4   -1,  l8.n7   1,
        l9.n4   -1,  l9.n9   1,
        l10.n5  -1,  l10.n6  1,
        l11.n6  -1,  l11.n9  1,
```

```
          l12.n6  -1,  l12.n12 1,
          l13.n6  -1,  l13.n13 1,
          l14.n7  -1,  l14.n8  1,
          l15.n7  -1,  l15.n9  1,
          l16.n9  -1,  l16.n10 1,
          l17.n9  -1,  l17.n14 1,
          l18.n11 -1,  l18.n10 1,
          l19.n12 -1,  l19.n13 1,
          l20.n14 -1,  l20.n13 1/

   CG(u,n) matrix of each generating unit connection's to node
     /u1.n1  1,  u2.n2 1,   u3.n3 1, u4.n3 1,    u5.n8 1,   u6.n13 1,
      u7.n6  1,  u8.n3 1,   u9.n14 1, u10.n4 1,    u11.n9 1/

   D(n,ss) total demand
       /n2.ss1 21.7, n3.ss1 94.2, n4.ss1 47.8, n5.ss1 7.6, n6.ss1 11.2, n9.ss»
1 29.5, n10.ss1 9, n11.ss1 3.5, n12.ss1 6.1, n13.ss1 13.5, n14.ss1 14.9/

   XX(l,ss) electrical reactance of line l
       /l1.ss1 0.05917, l2.ss1 0.22304, l3.ss1 0.19797, l4.ss1 0.17632, l5.ss»
1 0.17388, l6.ss1 0.17103, l7.ss1 0.04211, l8.ss1 0.20912, l9.ss1 0.55618, l1»
0.ss1 0.25202, l11.ss1 0.19890, l12.ss1 0.25581, l13.ss1 0.13027, l14.ss1 0.1»
7615, l15.ss1 0.11001, l16.ss1 0.08450, l17.ss1 0.27038, l18.ss1 0.19207, l19»
.ss1 0.19988, l20.ss1 0.34802/

   B(l,ss) electrical susceptance of line l

   cl(l,ss)  cost of removing a line
       /l1.ss1 5, l2.ss1 5, l3.ss1 5, l4.ss1 5, l5.ss1 5, l6.ss1 5, l7.ss1 5,»
 l8.ss1 5, l9.ss1 5, l10.ss1 5, l11.ss1 5, l12.ss1 5, l13.ss1 5, l14.ss1 5, l»
15.ss1 5, l16.ss1 5, l17.ss1 5, l18.ss1 5, l19.ss1 5, l20.ss1 5/

   nc(l,ss) operation status of a line during contingency
       /l1.ss1 1, l2.ss1 1, l3.ss1 1, l4.ss1 1, l5.ss1 1, l6.ss1 1, l7.ss1 1,»
 l8.ss1 1, l9.ss1 1, l10.ss1 1, l11.ss1 1, l12.ss1 1, l13.ss1 1, l14.ss1 1, l»
15.ss1 1, l16.ss1 1, l17.ss1 1, l18.ss1 1, l19.ss1 1, l20.ss1 1/

   prob(ss) probability of each scenario
       /ss1 1.0/

   num_units(p) number of units in portfolio p;

   num_units(p)=sum(u$owns(p,u),1);
   B(l,ss)=1/XX(l,ss)
   ;

* The following variables will need to be changed according to the
* number of actions per unit

Sets
   a possible actions for each unit 'a'
      /q1*q4/
   s enumeration of strategies for each portfolio n^a /s1*s4/
   k index of binary digits - b1 LSB - log2(a) - /w1*w4/

* For each portfolio the full list of potential strategies must be
* listed. Each strategy must specify one and only one action for each
* strategic unit

   sl(p,s,u,a) list of allowed strategies for each portfolio
   ps(p,s) list of strategies defined for each portfolio
```

```
   us(u) list of strategic generators
   un(u) list of non-strategic generators;

* Use this equation when portfolio p has only 1 unit
   sl(p,s,u,a)$(num_units(p)=1) = yes$(ord(s)=ord(a))*owns(p,u) ;

* The following equation only works when portfolio p has exactly 2 units
   sl(p,s,u,a)$(num_units(p)=2)
           = yes$((mod(ord(s)-1,4)=(ord(a)-1))*owns(p,u)*(mod(ord(u),2)=1)) +
             yes$((mod(trunc((ord(s)-1)/4),4)=(ord(a)-1))*owns(p,u)*(mod(ord(»
u),2)=0)) ;

   ps(p,s) = yes$(sum((u,a),sl(p,s,u,a))) ;
   us(u) = yes$(sum(p,owns(p,u))) ;
   un(u) = yes$(not us(u));
   alias(p,cp) ;
   alias(s,cs) ;


Parameters
   BigM a big number /999999/
   w0 additive offset for constant
   w(k) conversion factor from binary to production
   y(a,k) mapping from unit action to share of output ;

   y(a,k) = mod(trunc((ord(a)-1)/power(2,ord(k)-1)),2) ;

   w0 = power(2,-card(k));
   w(k) = power(2,-(card(k)-ord(k)+1)) ;


Table CT(n, l) transpose of the incidence matrix 'all data are logged and in »
csv format'
l1 l2 l3 l4 l5 l6 l7 l8 l9 l10 l11 l12 l13 l14 l15 l16 l17 l18 l19 l20

$ondelim
$include matrices_14.csv
$offdelim

Variables
   social_cost
   Fl0(l,ss)
   Fl0c(l,ss)
   theta0(n,ss)
   theta0c(n,ss)
   Fl(cp,cs,l,ss)
   Flc(cp,cs,l,ss)
   theta(cp,cs,n,ss)
   thetac(cp,cs,n,ss)
   pi0(p,ss) profit of portfolio p
   pi(cp,cs,p,ss) profit of portfolio p in case (cp cs)
   ;
Free variable
   rho0(n,ss)
   delta0(n,ss)
   rho(cp,cs,n,ss)
   delta(cp,cs,n,ss)
   ;
Positive Variables
   g0(u,ss)
   gh0(u,ss)
```

```
   z0(u,k,ss)
   g(cp,cs,u,ss)
   gh(cp,cs,u,ss)
   z(cp,cs,u,k,ss)

   lambdadown0(l,ss)
   mudown0(u,ss)
   betadown0(l,ss)
   alphadown0(l,ss)
   epsilondown0(l,ss)
   lambdaup0(l,ss)
   muup0(u,ss)
   betaup0(l,ss)
   alphaup0(l,ss)
   epsilonup0(l,ss)
   lambdadown(cp,cs,l,ss)
   mudown(cp,cs,u,ss)
   betadown(cp,cs,l,ss)
   alphadown(cp,cs,l,ss)
   epsilondown(cp,cs,l,ss)
   lambdaup(cp,cs,l,ss)
   muup(cp,cs,u,ss)
   betaup(cp,cs,l,ss)
   alphaup(cp,cs,l,ss)
   epsilonup(cp,cs,l,ss)
   ;
Binary Variables
   nl(l,ss)
   x0(u,k,ss)
   x(cp,cs,u,k,ss)
   b_lambdadown0(l,ss)
   b_mudown0(u,ss)
   b_betadown0(l,ss)
   b_alphadown0(l,ss)
   b_epsilondown0(l,ss)
   b_lambdaup0(l,ss)
   b_muup0(u,ss)
   b_betaup0(l,ss)
   b_alphaup0(l,ss)
   b_epsilonup0(l,ss)
   b_lambdadown(cp,cs,l,ss)
   b_mudown(cp,cs,u,ss)
   b_betadown(cp,cs,l,ss)
   b_alphadown(cp,cs,l,ss)
   b_epsilondown(cp,cs,l,ss)
   b_lambdaup(cp,cs,l,ss)
   b_muup(cp,cs,u,ss)
   b_betaup(cp,cs,l,ss)
   b_alphaup(cp,cs,l,ss)
   b_epsilonup(cp,cs,l,ss)
   ;

*Objective function - social cost

Equations
   obj
;

obj ..               social_cost =e= (sum{ss,prob(ss)*sum(u,cost(u)*g0(u,ss))»
}+sum{(l,ss),cl(l,ss)*(1-nl(l,ss))}) ;          ;

```

```
***************************************************************************»
*****************************************************************
*Optimal dispatch for the candidate WNE
***************************************************************************»
*****************************************************************

Equations

   e01_0(u,ss)   volume of capacity that its offer to the market
   e02_0(p,ss)   profit of a portafolio
   eA_0(u,k,ss)  linearization of equation gh0
   eB_0(u,k,ss)  linearization of equation gh0
   eC_0(u,k,ss)  linearization of equation gh0
   eD_0(u,k,ss)  linearization of equation gh0
;
e01_0(u,ss)   ..      gh0(u,ss) =e= (w0 + sum(k,w(k)*x0(u,k,ss)))*cap(u,ss);
e02_0(p,ss)   ..      pi0(p,ss) =e= sum(u$owns(p,u),(muup0(u,ss)*w0 + sum(k,w»
(k)*z0(u,k,ss)))*cap(u,ss));
eA_0(us,k,ss) ..      z0(us,k,ss)-muup0(us,ss) =l= BigM*(1-x0(us,k,ss));
eB_0(us,k,ss) ..      z0(us,k,ss)-muup0(us,ss) =g= -BigM*(1-x0(us,k,ss));
eC_0(us,k,ss) ..      z0(us,k,ss) =l= BigM*x0(us,k,ss);
eD_0(u,k,ss)$un(u) .. x0(u,k,ss) =e= 1;

* KKT conditions
*  Primal
*    DCOPF problem with optimal transmission switching

Equations

   e2_0(u,ss)  maximun generation
   e5_0(l,ss)  maximun power flow across line limit
   e6_0(l,ss)  minimun power flow across line limit
   e7_0(n,ss)  power balance at each node
   e8_0(l,ss)  kirchhoff's laws
   e9_0(l,ss)  kirchhoff's laws

;
e2_0(u,ss) ..         g0(u,ss)=l= gh0(u,ss);
e5_0(l,ss) ..         Fl0(l,ss) =l= F(l,ss)*nl(l,ss);
e6_0(l,ss) ..         Fl0(l,ss) =g= -F(l,ss)*nl(l,ss);
e7_0(n,ss) ..         sum(l, Fl0(l,ss)*C(l,n))+(sum(u$nd(n,u),g0(u,ss))-D(n,s»
s))=e=0;
e8_0(l,ss) ..         sum{n, B(l,ss)*C(l,n)*theta0(n,ss)}- Fl0(l,ss)+(1-nl(l,»
ss))*BigM =g= 0;
e9_0(l,ss) ..         sum{n, B(l,ss)*C(l,n)*theta0(n,ss)}- Fl0(l,ss)-(1-nl(l,»
ss))*BigM =l= 0;

*    DCOPF optimal transmission switching with contingency analysis

Equations

   e12_0(l,ss) maximun power flow across line limit for contingency c
   e13_0(l,ss) minimun power flow across line limit for contingency c
   e14_0(n,ss) power balance at each node for contingency c
   e15_0(l,ss) kirchhoff's laws for contingency c
   e16_0(l,ss) kirchhoff's laws for contingency c
;
e12_0(l,ss) ..        Fl0c(l,ss) =l= Fc(l,ss)*nl(l,ss)*nc(l,ss);
e13_0(l,ss) ..        Fl0c(l,ss) =g= -Fc(l,ss)*nl(l,ss)*nc(l,ss);
e14_0(n,ss) ..        sum(l, Fl0c(l,ss)*C(l,n))+(sum(u$nd(n,u),g0(u,ss))-D(n,»
ss))=e=0;
```

```
e15_0(l,ss) ..        sum{n, B(l,ss)*C(l,n)*theta0c(n,ss)}- Fl0c(l,ss)+(2-nl(»
l,ss)-nc(l,ss))*BigM =g= 0;
e16_0(l,ss) ..        sum{n, B(l,ss)*C(l,n)*theta0c(n,ss)}- Fl0c(l,ss)-(2-nl(»
l,ss)-nc(l,ss))*BigM =l= 0;

*  Stationary conditions

Equations

   e17_0(u,ss) derivate of Lagrangian respect to the generation of each unit
   e18_0(n,ss) derivate of Lagrangian respect to the voltage angle of each no»
de
   e19_0(l,ss) derivate of Lagrangian respect to the power flow of each line
   e20_0(n,ss) derivate of Lagrangian respect to the voltage angle of each no»
de during contingency c
   e21_0(l,ss) derivate of Lagrangian respect to the power flow of each line »
during contingency c
;
e17_0(u,ss) ..        -cost(u)+sum(n, rho0(n,ss)*CG(u,n))-mudown0(u,ss)+muup0»
(u,ss)+sum(n, delta0(n,ss)*CG(u,n))=e=0;
e18_0(n,ss) ..        sum{l, CT(n,l)*betadown0(l,ss)*B(l,ss)}-sum{l, CT(n,l)*»
betaup0(l,ss)*B(l,ss)}=e=0;
e19_0(l,ss) ..        sum(n, rho0(n,ss)*C(l,n))-lambdadown0(l,ss)+lambdaup0(l»
,ss)+betadown0(l,ss)-betaup0(l,ss)=e=0;
e20_0(n,ss) ..        sum{l, CT(n,l)*epsilondown0(l,ss)*B(l,ss)}-sum{l, CT(n,»
l)*epsilonup0(l,ss)*B(l,ss)}=e=0;
e21_0(l,ss) ..        sum(n, delta0(n,ss)*C(l,n))-alphadown0(l,ss)+alphaup0(l»
,ss)+epsilondown0(l,ss)-epsilonup0(l,ss)=e=0;

*  Complementary slackness conditions
*  Reformulating as a Mix Integer Linear Programming Problem (MILP)

Equations

   e36_0(l,ss) Lower boundary of the capacity constraints on transmission lin»
es as MILP
   e37_0(l,ss) Lower boundary of the capacity constraints on transmission lin»
es as MILP
   e38_0(l,ss) Lower boundary of the capacity constraints on transmission lin»
es as MILP
   e39_0(l,ss) Lower boundary of the capacity constraints on transmission lin»
es as MILP
   e40_0(l,ss) Upper boundary of the capacity constraints on transmission lin»
es as MILP
   e41_0(l,ss) Upper boundary of the capacity constraints on transmission lin»
es as MILP
   e42_0(l,ss) Upper boundary of the capacity constraints on transmission lin»
es as MILP
   e43_0(l,ss) Upper boundary of the capacity constraints on transmission lin»
es as MILP
   e44_0(u,ss) Lower boundary of the capacity constraints on each generating »
units as MILP
   e45_0(u,ss) Lower boundary of the capacity constraints on each generating »
units as MILP
   e46_0(u,ss) Lower boundary of the capacity constraints on each generating »
units as MILP
   e47_0(u,ss) Lower boundary of the capacity constraints on each generating »
units as MILP
   e48_0(u,ss) Upper boundary of the capacity constraints on each generating »
units as MILP
   e49_0(u,ss) Upper boundary of the capacity constraints on each generating »
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    units as MILP
       e50_0(u,ss) Upper boundary of the capacity constraints on each generating »
    units as MILP
       e51_0(u,ss) Upper boundary of the capacity constraints on each generating »
    units as MILP
       e52_0(l,ss) Lower boundary of constraints on kirchhoff's laws as MILP
       e53_0(l,ss) Lower boundary of constraints on kirchhoff's laws as MILP
       e54_0(l,ss) Lower boundary of constraints on kirchhoff's laws as MILP
       e55_0(l,ss) Lower boundary of constraints on kirchhoff's laws as MILP
       e56_0(l,ss) Upper boundary of constraints on kirchhoff's laws as MILP
       e57_0(l,ss) Upper boundary of constraints on kirchhoff's laws as MILP
       e58_0(l,ss) Upper boundary of constraints on kirchhoff's laws as MILP
       e59_0(l,ss) Upper boundary of constraints on kirchhoff's laws as MILP
       e68_0(l,ss) Lower boundary of the capacity constraints on transmission lin»
    es as MILP for contingency c
       e69_0(l,ss) Lower boundary of the capacity constraints on transmission lin»
    es as MILP for contingency c
       e70_0(l,ss) Lower boundary of the capacity constraints on transmission lin»
    es as MILP for contingency c
       e71_0(l,ss) Lower boundary of the capacity constraints on transmission lin»
    es as MILP for contingency c
       e72_0(l,ss) Upper boundary of the capacity constraints on transmission lin»
    es as MILP for contingency c
       e73_0(l,ss) Upper boundary of the capacity constraints on transmission lin»
    es as MILP for contingency c
       e74_0(l,ss) Upper boundary of the capacity constraints on transmission lin»
    es as MILP for contingency c
       e75_0(l,ss) Upper boundary of the capacity constraints on transmission lin»
    es as MILP for contingency c
       e76_0(l,ss) Lower boundary of constraints on kirchhoff's laws as MILP for »
    contingency c
       e77_0(l,ss) Lower boundary of constraints on kirchhoff's laws as MILP for »
    contingency c
       e78_0(l,ss) Lower boundary of constraints on kirchhoff's laws as MILP for »
    contingency c
       e79_0(l,ss) Lower boundary of constraints on kirchhoff's laws as MILP for »
    contingency c
       e80_0(l,ss) Upper boundary of constraints on kirchhoff's laws as MILP for »
    contingency c
       e81_0(l,ss) Upper boundary of constraints on kirchhoff's laws as MILP for »
    contingency c
       e82_0(l,ss) Upper boundary of constraints on kirchhoff's laws as MILP for »
    contingency c
       e83_0(l,ss) Upper boundary of constraints on kirchhoff's laws as MILP for »
    contingency c
       ;

e36_0(l,ss) ..        -bigM*(1-b_lambdadown0(l,ss))=l= (-F(l,ss)*nl(l,ss)-Fl0»
(l,ss));
e37_0(l,ss) ..        (-F(l,ss)*nl(l,ss)-Fl0(l,ss))=l= bigM*(1-b_lambdadown0(»
l,ss));
e38_0(l,ss) ..        -bigM*(b_lambdadown0(l,ss))=l=lambdadown0(l,ss);
e39_0(l,ss) ..        lambdadown0(l,ss)=l=bigM*(b_lambdadown0(l,ss));
e40_0(l,ss) ..        -bigM*(1-b_lambdaup0(l,ss))=l=(Fl0(l,ss)-F(l,ss)*nl(l,s»
s));
e41_0(l,ss) ..        (Fl0(l,ss)-F(l,ss)*nl(l,ss))=l= bigM*(1-b_lambdaup0(l,s»
s));
e42_0(l,ss) ..        -bigM*(b_lambdaup0(l,ss))=l=lambdaup0(l,ss);
e43_0(l,ss) ..        lambdaup0(l,ss)=l=bigM*(b_lambdaup0(l,ss));

e44_0(u,ss) ..        -bigM*(1-b_mudown0(u,ss))=l= (-g0(u,ss));
```

```
e45_0(u,ss) ..        (-g0(u,ss))=l= bigM*(1-b_mudown0(u,ss));
e46_0(u,ss) ..        -bigM*(b_mudown0(u,ss))=l=mudown0(u,ss);
e47_0(u,ss) ..        mudown0(u,ss)=l= bigM*(b_mudown0(u,ss));
e48_0(u,ss) ..        -bigM*(1-b_muup0(u,ss))=l=(g0(u,ss)-gh0(u,ss));
e49_0(u,ss) ..        (g0(u,ss)-gh0(u,ss))=l= bigM*(1-b_muup0(u,ss));
e50_0(u,ss) ..        -bigM*(b_muup0(u,ss))=l=muup0(u,ss);
e51_0(u,ss) ..        muup0(u,ss)=l=bigM*(b_muup0(u,ss));

e52_0(l,ss) ..        -bigM*(1-b_betadown0(l,ss))=l= sum{n,(-B(l,ss))*C(l,n)*theta0(n,ss)}+ Fl0(l,ss)-(1-nl(l,ss))*BigM;
e53_0(l,ss) ..        sum{n,(-B(l,ss))*C(l,n)*theta0(n,ss)}+ Fl0(l,ss)-(1-nl(l,ss))*BigM =l= bigM*(1-b_betadown0(l,ss));
e54_0(l,ss) ..        -bigM*(b_betadown0(l,ss))=l=betadown0(l,ss);
e55_0(l,ss) ..        betadown0(l,ss)=l=bigM*(b_betadown0(l,ss));
e56_0(l,ss) ..        -bigM*(1-b_betaup0(l,ss))=l= sum{n,B(l,ss)*C(l,n)*theta0(n,ss)}- Fl0(l,ss)-(1-nl(l,ss))*BigM;
e57_0(l,ss) ..        sum{n,B(l,ss)*C(l,n)*theta0(n,ss)}- Fl0(l,ss)-(1-nl(l,ss))*BigM =l= bigM*(1-b_betaup0(l,ss));
e58_0(l,ss) ..        -bigM*(b_betaup0(l,ss))=l=betaup0(l,ss);
e59_0(l,ss) ..        betaup0(l,ss)=l=bigM*(b_betaup0(l,ss));

e68_0(l,ss) ..        -bigM*(1-b_alphadown0(l,ss))=l= (-Fc(l,ss)*nl(l,ss)*nc(l,ss)-Fl0c(l,ss));
e69_0(l,ss) ..        (-Fc(l,ss)*nl(l,ss)*nc(l,ss)-Fl0c(l,ss))=l= bigM*(1-b_alphadown0(l,ss));
e70_0(l,ss) ..        -bigM*(b_alphadown0(l,ss))=l=alphadown0(l,ss);
e71_0(l,ss) ..        alphadown0(l,ss)=l=bigM*(b_alphadown0(l,ss));
e72_0(l,ss) ..        -bigM*(1-b_alphaup0(l,ss))=l=(Fl0c(l,ss)-Fc(l,ss)*nl(l,ss)*nc(l,ss));
e73_0(l,ss) ..        (Fl0c(l,ss)-Fc(l,ss)*nl(l,ss)*nc(l,ss))=l= bigM*(1-b_alphaup0(l,ss));
e74_0(l,ss) ..        -bigM*(b_alphaup0(l,ss))=l=alphaup0(l,ss);
e75_0(l,ss) ..        alphaup0(l,ss)=l=bigM*(b_alphaup0(l,ss));

e76_0(l,ss) ..        -bigM*(1-b_epsilondown0(l,ss))=l= sum{n,(-B(l,ss))*C(l,n)*theta0c(n,ss)}+ Fl0c(l,ss)-(2-nl(l,ss)-nc(l,ss))*BigM ;
e77_0(l,ss) ..        sum{n,(-B(l,ss))*C(l,n)*theta0c(n,ss)}+ Fl0c(l,ss)-(2-nl(l,ss)-nc(l,ss))*BigM =l= bigM*(1-b_epsilondown0(l,ss));
e78_0(l,ss) ..        -bigM*(b_epsilondown0(l,ss))=l=epsilondown0(l,ss);
e79_0(l,ss) ..        epsilondown0(l,ss)=l=bigM*(b_epsilondown0(l,ss));
e80_0(l,ss) ..        -bigM*(1-b_epsilonup0(l,ss))=l= sum{n,B(l,ss)*C(l,n)*theta0c(n,ss)}- Fl0c(l,ss)-(2-nl(l,ss)-nc(l,ss))*BigM;
e81_0(l,ss) ..        sum{n,B(l,ss)*C(l,n)*theta0c(n,ss)}- Fl0c(l,ss)-(2-nl(l,ss)-nc(l,ss))*BigM =l= bigM*(1-b_epsilonup0(l,ss));
e82_0(l,ss) ..        -bigM*(b_epsilonup0(l,ss))=l=epsilonup0(l,ss);
e83_0(l,ss) ..        epsilonup0(l,ss)=l= bigM*(b_epsilonup0(l,ss));

********************************************************************************************************************************************
*Optimal dispatch for the alternative case p s
********************************************************************************************************************************************

Equations

   e01(cp,cs,u,ss)   volume of capacity that its offer to the market
   e02(cp,cs,p,ss)   profit of a portafolio
   eA(cp,cs,u,k,ss)  linearization of equation gh0
   eB(cp,cs,u,k,ss)  linearization of equation gh0
   eC(cp,cs,u,k,ss)  linearization of equation gh0
   eD(cp,cs,u,k,ss)  linearization of equation gh0
```

```
;
e01(cp,cs,u,ss)   ..      gh(cp,cs,u,ss) =e= (w0 + sum(k,w(k)*x(cp,cs,u,k,ss)»
))*cap(u,ss);
e02(cp,cs,p,ss)   ..      pi(cp,cs,p,ss) =e= sum(u$owns(p,u),(muup(cp,cs,u,ss»
)*w0 + sum(k,w(k)*z(cp,cs,u,k,ss)))*cap(u,ss));
eA(cp,cs,us,k,ss) ..      z(cp,cs,us,k,ss)-muup(cp,cs,us,ss) =l= BigM*(1-x(cp»
,cs,us,k,ss));
eB(cp,cs,us,k,ss) ..      z(cp,cs,us,k,ss) =l= BigM*x(cp,cs,us,k,ss);
eC(cp,cs,us,k,ss) ..      z(cp,cs,us,k,ss)-muup(cp,cs,us,ss) =g= -BigM*(1-x(c»
p,cs,us,k,ss));
eD(cp,cs,u,k,ss)  ..      x(cp,cs,u,k,ss) =e= x0(u,k,ss)$(not owns(cp,u)) + s»
um(a,y(a,k)$sl(cp,cs,u,a))$owns(cp,u);

* KKT conditions
*  Primal
*    DCOPF problem with optimal transmission switching

Equations

   e2(cp,cs,u,ss)  maximun generation
   e5(cp,cs,l,ss)  maximun power flow across line limit
   e6(cp,cs,l,ss)  minimun power flow across line limit
   e7(cp,cs,n,ss)  power balance at each node
   e8(cp,cs,l,ss)  kirchhoff's laws
   e9(cp,cs,l,ss)  kirchhoff's laws
;
e2(cp,cs,u,ss) ..         g(cp,cs,u,ss)=l= gh(cp,cs,u,ss) ;
e5(cp,cs,l,ss) ..         Fl(cp,cs,l,ss) =l= F(l,ss)*nl(l,ss);
e6(cp,cs,l,ss) ..         Fl(cp,cs,l,ss) =g= -F(l,ss)*nl(l,ss);
e7(cp,cs,n,ss) ..         sum(l, Fl(cp,cs,l,ss)*C(l,n))+(sum(u$nd(n,u),g(cp,c»
s,u,ss))-D(n,ss))=e=0;
e8(cp,cs,l,ss) ..         sum{n, B(l,ss)*C(l,n)*theta(cp,cs,n,ss)}- Fl(cp,cs,»
l,ss)+(1-nl(l,ss))*BigM =g= 0;
e9(cp,cs,l,ss) ..         sum{n, B(l,ss)*C(l,n)*theta(cp,cs,n,ss)}- Fl(cp,cs,»
l,ss)-(1-nl(l,ss))*BigM =l= 0;

*    DCOPF optimal transmission switching with contingency analysis

Equations

   e12(cp,cs,l,ss) maximun power flow across line limit for contingency c
   e13(cp,cs,l,ss) minimun power flow across line limit for contingency c
   e14(cp,cs,n,ss) power balance at each node for contingency c
   e15(cp,cs,l,ss) kirchhoff's laws for contingency c
   e16(cp,cs,l,ss) kirchhoff's laws for contingency c
;
e12(cp,cs,l,ss) ..        Flc(cp,cs,l,ss) =l= Fc(l,ss)*nl(l,ss)*nc(l,ss);
e13(cp,cs,l,ss) ..        Flc(cp,cs,l,ss) =g= -Fc(l,ss)*nl(l,ss)*nc(l,ss);
e14(cp,cs,n,ss) ..        sum(l, Flc(cp,cs,l,ss)*C(l,n))+(sum(u$nd(n,u),g(cp,»
cs,u,ss))-D(n,ss))=e=0;
e15(cp,cs,l,ss) ..        sum{n, B(l,ss)*C(l,n)*thetac(cp,cs,n,ss)}- Flc(cp,c»
s,l,ss)+(2-nl(l,ss)-nc(l,ss))*BigM =g= 0;
e16(cp,cs,l,ss) ..        sum{n, B(l,ss)*C(l,n)*thetac(cp,cs,n,ss)}- Flc(cp,c»
s,l,ss)-(2-nl(l,ss)-nc(l,ss))*BigM =l= 0;

*  Stationary conditions

Equations

   e17(cp,cs,u,ss) derivate of Lagrangian respect to the generation of each u»
nit
```

```
   e18(cp,cs,n,ss) derivate of Lagrangian respect to the voltage angle of eac»
h node
   e19(cp,cs,l,ss) derivate of Lagrangian respect to the power flow of each l»
ine
   e20(cp,cs,n,ss) derivate of Lagrangian respect to the voltage angle of eac»
h node during contingency c
   e21(cp,cs,l,ss) derivate of Lagrangian respect to the power flow of each l»
ine during contingency c
;
e17(cp,cs,u,ss) ..        -cost(u)+sum(n, rho(cp,cs,n,ss)*CG(u,n))-mudown(cp,»
cs,u,ss)+muup(cp,cs,u,ss)+sum(n, delta(cp,cs,n,ss)*CG(u,n))=e=0;
e18(cp,cs,n,ss) ..        sum{l, CT(n,l)*betadown(cp,cs,l,ss)*B(l,ss)}-sum{l,»
 CT(n,l)*betaup(cp,cs,l,ss)*B(l,ss)}=e=0;
e19(cp,cs,l,ss) ..        sum(n, rho(cp,cs,n,ss)*C(l,n))-lambdadown(cp,cs,l,s»
s)+lambdaup(cp,cs,l,ss)+betadown(cp,cs,l,ss)-betaup(cp,cs,l,ss)=e=0;
e20(cp,cs,n,ss) ..        sum{l, CT(n,l)*epsilondown(cp,cs,l,ss)*B(l,ss)}-sum»
{l, CT(n,l)*epsilonup(cp,cs,l,ss)*B(l,ss)}=e=0;
e21(cp,cs,l,ss) ..        sum(n, delta(cp,cs,n,ss)*C(l,n))-alphadown(cp,cs,l,»
ss)+alphaup(cp,cs,l,ss)+epsilondown(cp,cs,l,ss)-epsilonup(cp,cs,l,ss)=e=0;

*  Complementary slackness conditions
*  Reformulating as a Mix Integer Linear Programming Problem (MILP)

Equations

   e36(cp,cs,l,ss) Lower boundary of the capacity constraints on transmission»
 lines as MILP
   e37(cp,cs,l,ss) Lower boundary of the capacity constraints on transmission»
 lines as MILP
   e38(cp,cs,l,ss) Lower boundary of the capacity constraints on transmission»
 lines as MILP
   e39(cp,cs,l,ss) Lower boundary of the capacity constraints on transmission»
 lines as MILP
   e40(cp,cs,l,ss) Upper boundary of the capacity constraints on transmission»
 lines as MILP
   e41(cp,cs,l,ss) Upper boundary of the capacity constraints on transmission»
 lines as MILP
   e42(cp,cs,l,ss) Upper boundary of the capacity constraints on transmission»
 lines as MILP
   e43(cp,cs,l,ss) Upper boundary of the capacity constraints on transmission»
 lines as MILP
   e44(cp,cs,u,ss) Lower boundary of the capacity constraints on each generat»
ing units as MILP
   e45(cp,cs,u,ss) Lower boundary of the capacity constraints on each generat»
ing units as MILP
   e46(cp,cs,u,ss) Lower boundary of the capacity constraints on each generat»
ing units as MILP
   e47(cp,cs,u,ss) Lower boundary of the capacity constraints on each generat»
ing units as MILP
   e48(cp,cs,u,ss) Upper boundary of the capacity constraints on each generat»
ing units as MILP
   e49(cp,cs,u,ss) Upper boundary of the capacity constraints on each generat»
ing units as MILP
   e50(cp,cs,u,ss) Upper boundary of the capacity constraints on each generat»
ing units as MILP
   e51(cp,cs,u,ss) Upper boundary of the capacity constraints on each generat»
ing units as MILP
   e52(cp,cs,l,ss) Lower boundary of constraints on kirchhoff's laws as MILP
   e53(cp,cs,l,ss) Lower boundary of constraints on kirchhoff's laws as MILP
   e54(cp,cs,l,ss) Lower boundary of constraints on kirchhoff's laws as MILP
   e55(cp,cs,l,ss) Lower boundary of constraints on kirchhoff's laws as MILP
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   e56(cp,cs,l,ss) Upper boundary of constraints on kirchhoff's laws as MILP
   e57(cp,cs,l,ss) Upper boundary of constraints on kirchhoff's laws as MILP
   e58(cp,cs,l,ss) Upper boundary of constraints on kirchhoff's laws as MILP
   e59(cp,cs,l,ss) Upper boundary of constraints on kirchhoff's laws as MILP
   e68(cp,cs,l,ss) Lower boundary of the capacity constraints on transmission»
  lines as MILP for contingency c
   e69(cp,cs,l,ss) Lower boundary of the capacity constraints on transmission»
  lines as MILP for contingency c
   e70(cp,cs,l,ss) Lower boundary of the capacity constraints on transmission»
  lines as MILP for contingency c
   e71(cp,cs,l,ss) Lower boundary of the capacity constraints on transmission»
  lines as MILP for contingency c
   e72(cp,cs,l,ss) Upper boundary of the capacity constraints on transmission»
  lines as MILP for contingency c
   e73(cp,cs,l,ss) Upper boundary of the capacity constraints on transmission»
  lines as MILP for contingency c
   e74(cp,cs,l,ss) Upper boundary of the capacity constraints on transmission»
  lines as MILP for contingency c
   e75(cp,cs,l,ss) Upper boundary of the capacity constraints on transmission»
  lines as MILP for contingency c
   e76(cp,cs,l,ss) Lower boundary of constraints on kirchhoff's laws as MILP »
 for contingency c
   e77(cp,cs,l,ss) Lower boundary of constraints on kirchhoff's laws as MILP »
 for contingency c
   e78(cp,cs,l,ss) Lower boundary of constraints on kirchhoff's laws as MILP »
 for contingency c
   e79(cp,cs,l,ss) Lower boundary of constraints on kirchhoff's laws as MILP »
 for contingency c
   e80(cp,cs,l,ss) Upper boundary of constraints on kirchhoff's laws as MILP »
 for contingency c
   e81(cp,cs,l,ss) Upper boundary of constraints on kirchhoff's laws as MILP »
 for contingency c
   e82(cp,cs,l,ss) Upper boundary of constraints on kirchhoff's laws as MILP »
 for contingency c
   e83(cp,cs,l,ss) Upper boundary of constraints on kirchhoff's laws as MILP »
 for contingency c
   ;

e36(cp,cs,l,ss) ..       -bigM*(1-b_lambdadown(cp,cs,l,ss))=l= (-F(l,ss)*nl(»
 l,ss)-Fl(cp,cs,l,ss));
e37(cp,cs,l,ss) ..       (-F(l,ss)*nl(l,ss)-Fl(cp,cs,l,ss))=l= bigM*(1-b_lam»
 bdadown(cp,cs,l,ss));
e38(cp,cs,l,ss) ..       -bigM*(b_lambdadown(cp,cs,l,ss))=l=lambdadown(cp,cs»
 ,l,ss);
e39(cp,cs,l,ss) ..       lambdadown(cp,cs,l,ss)=l=bigM*(b_lambdadown(cp,cs,l»
 ,ss));
e40(cp,cs,l,ss) ..       -bigM*(1-b_lambdaup(cp,cs,l,ss))=l=(Fl(cp,cs,l,ss)-»
 F(l,ss)*nl(l,ss));
e41(cp,cs,l,ss) ..       (Fl(cp,cs,l,ss)-F(l,ss)*nl(l,ss))=l= bigM*(1-b_lamb»
 daup(cp,cs,l,ss));
e42(cp,cs,l,ss) ..       -bigM*(b_lambdaup(cp,cs,l,ss))=l=lambdaup(cp,cs,l,s»
 s);
e43(cp,cs,l,ss) ..       lambdaup(cp,cs,l,ss)=l=bigM*(b_lambdaup(cp,cs,l,ss)»
 );

e44(cp,cs,u,ss) ..       -bigM*(1-b_mudown(cp,cs,u,ss))=l= (-g(cp,cs,u,ss));
e45(cp,cs,u,ss) ..       (-g(cp,cs,u,ss))=l= bigM*(1-b_mudown(cp,cs,u,ss));
e46(cp,cs,u,ss) ..       -bigM*(b_mudown(cp,cs,u,ss))=l=mudown(cp,cs,u,ss);
e47(cp,cs,u,ss) ..       mudown(cp,cs,u,ss)=l= bigM*(b_mudown(cp,cs,u,ss));
e48(cp,cs,u,ss) ..       -bigM*(1-b_muup(cp,cs,u,ss))=l=(g(cp,cs,u,ss)-gh(cp»
 ,cs,u,ss));
```

```
e49(cp,cs,u,ss) ..        (g(cp,cs,u,ss)-gh(cp,cs,u,ss))=l= bigM*(1-b_muup(cp»
,cs,u,ss));
e50(cp,cs,u,ss) ..        -bigM*(b_muup(cp,cs,u,ss))=l=muup(cp,cs,u,ss);
e51(cp,cs,u,ss) ..        muup(cp,cs,u,ss)=l=bigM*(b_muup(cp,cs,u,ss));

e52(cp,cs,l,ss) ..        -bigM*(1-b_betadown(cp,cs,l,ss))=l= sum{n,(-B(l,ss)»
)*C(l,n)*theta(cp,cs,n,ss)}+ Fl(cp,cs,l,ss)-(1-nl(l,ss))*BigM;
e53(cp,cs,l,ss) ..        sum{n,(-B(l,ss))*C(l,n)*theta(cp,cs,n,ss)}+ Fl(cp,c»
s,l,ss)-(1-nl(l,ss))*BigM =l= bigM*(1-b_betadown(cp,cs,l,ss));
e54(cp,cs,l,ss) ..        -bigM*(b_betadown(cp,cs,l,ss))=l=betadown(cp,cs,l,s»
s);
e55(cp,cs,l,ss) ..        betadown(cp,cs,l,ss)=l=bigM*(b_betadown(cp,cs,l,ss)»
);
e56(cp,cs,l,ss) ..        -bigM*(1-b_betaup(cp,cs,l,ss))=l= sum{n,B(l,ss)*C(l»
,n)*theta(cp,cs,n,ss)}- Fl(cp,cs,l,ss)-(1-nl(l,ss))*BigM;
e57(cp,cs,l,ss) ..        sum{n,B(l,ss)*C(l,n)*theta(cp,cs,n,ss)}- Fl(cp,cs,l»
,ss)-(1-nl(l,ss))*BigM =l= bigM*(1-b_betaup(cp,cs,l,ss));
e58(cp,cs,l,ss) ..        -bigM*(b_betaup(cp,cs,l,ss))=l=betaup(cp,cs,l,ss);
e59(cp,cs,l,ss) ..        betaup(cp,cs,l,ss)=l=bigM*(b_betaup(cp,cs,l,ss));
e68(cp,cs,l,ss) ..        -bigM*(1-b_alphadown(cp,cs,l,ss))=l= (-Fc(l,ss)*nl(»
l,ss)*nc(l,ss)-Flc(cp,cs,l,ss));
e69(cp,cs,l,ss) ..        (-Fc(l,ss)*nl(l,ss)*nc(l,ss)-Flc(cp,cs,l,ss))=l= bi»
gM*(1-b_alphadown(cp,cs,l,ss));
e70(cp,cs,l,ss) ..        -bigM*(b_alphadown(cp,cs,l,ss))=l=alphadown(cp,cs,l»
,ss);
e71(cp,cs,l,ss) ..        alphadown(cp,cs,l,ss)=l=bigM*(b_alphadown(cp,cs,l,s»
s));
e72(cp,cs,l,ss) ..        -bigM*(1-b_alphaup(cp,cs,l,ss))=l=(Flc(cp,cs,l,ss)-»
Fc(l,ss)*nl(l,ss)*nc(l,ss));
e73(cp,cs,l,ss) ..        (Flc(cp,cs,l,ss)-Fc(l,ss)*nl(l,ss)*nc(l,ss))=l= big»
M*(1-b_alphaup(cp,cs,l,ss));
e74(cp,cs,l,ss) ..        -bigM*(b_alphaup(cp,cs,l,ss))=l=alphaup(cp,cs,l,ss)»
;
e75(cp,cs,l,ss) ..        alphaup(cp,cs,l,ss)=l=bigM*(b_alphaup(cp,cs,l,ss));

e76(cp,cs,l,ss) ..        -bigM*(1-b_epsilondown(cp,cs,l,ss))=l= sum{n,(-B(l,»
ss))*C(l,n)*thetac(cp,cs,n,ss)}+ Flc(cp,cs,l,ss)-(2-nl(l,ss)-nc(l,ss))*BigM ;
e77(cp,cs,l,ss) ..        sum{n,(-B(l,ss))*C(l,n)*thetac(cp,cs,n,ss)}+ Flc(cp»
,cs,l,ss)-(2-nl(l,ss)-nc(l,ss))*BigM =l= bigM*(1-b_epsilondown(cp,cs,l,ss));
e78(cp,cs,l,ss) ..        -bigM*(b_epsilondown(cp,cs,l,ss))=l=epsilondown(cp,»
cs,l,ss);
e79(cp,cs,l,ss) ..        epsilondown(cp,cs,l,ss)=l=bigM*(b_epsilondown(cp,cs»
,l,ss));
e80(cp,cs,l,ss) ..        -bigM*(1-b_epsilonup(cp,cs,l,ss))=l= sum{n,B(l,ss)*»
C(l,n)*thetac(cp,cs,n,ss)}- Flc(cp,cs,l,ss)-(2-nl(l,ss)-nc(l,ss))*BigM;
e81(cp,cs,l,ss) ..        sum{n,B(l,ss)*C(l,n)*thetac(cp,cs,n,ss)}- Flc(cp,cs»
,l,ss)-(2-nl(l,ss)-nc(l,ss))*BigM =l= bigM*(1-b_epsilonup(cp,cs,l,ss));
e82(cp,cs,l,ss) ..        -bigM*(b_epsilonup(cp,cs,l,ss))=l=epsilonup(cp,cs,l»
,ss);
e83(cp,cs,l,ss) ..        epsilonup(cp,cs,l,ss)=l= bigM*(b_epsilonup(cp,cs,l,»
ss));
*
* These equations ensure that the selected strategy combination is a NE
*
Equation
   epi(cp,cs,ss)

;
epi(cp,cs,ss)   ..        pi0(cp,ss)=g=pi(cp,cs,cp,ss);

Option rmip=cplex;
```

```
Model LinearNE /all/ ;

Solve LinearNE using rmip maximizing social_cost ;
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